ADVERSE SELECTION AND AUCTION DESIGN FOR INTERNET DISPLAY ADVERTISING

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1

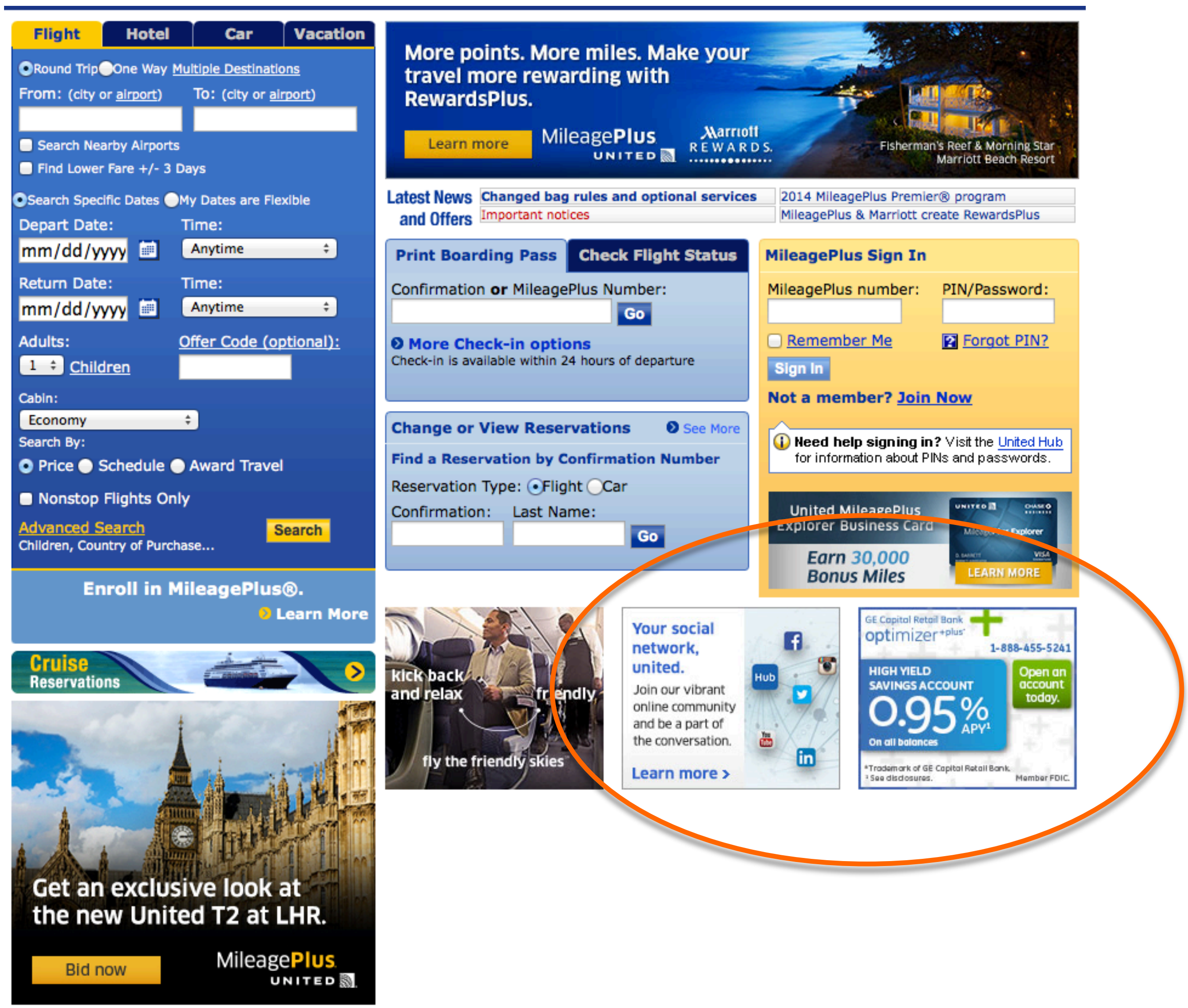
Old Advertisers & New

"Half the money I spend on advertising is wasted; the trouble is, I don't know which half."

- John Wanamaker, Advertising pioneer

Old-Fashioned "Brand" Ads

New-Fashioned "Performance" Ads



Display Advertisement Types

Brand Ads

- Goal: reach & repetition
  - For awareness and image
- Common Characteristics
  - Targeted to a large group
  - Large number of Impressions
  - Guaranteed delivery
- Sample Advertisers
  - Ford (weekend auto sale)
  - Disney (movie openings)
  - Shopping Center (location)

Performance Ads

- Goal: measurable action now
  - Click, fill form, or buy.
- Common Characteristics
  - Targeted to an individual
  - Smaller number of impressions
  - Sell individual impressions
- Sample Advertisers
  - Amazon (re-targeting)
  - Hertz (car rental)
  - Quicken mortgage (refinance)

Danger of Adverse Selection

Brand Advertisers

- Mostly buy large numbers of impressions.
- Receive deferred, aggregated data about performance of the whole ad campaign
- Cannot easily distinguish low-performing ads and publishers

Performance Advertisers

- Mostly select individual impressions using private cookies.
- Receive immediate, detailed data about the performance of individual ads
- Can quickly identify low-performing ads and publishers

If brand and performance advertisers' values are "positively correlated," then brand advertisers may suffer adverse selection.

Matching with Adverse Selection

Modeling the problem

Model

- There are $N+1$ advertisers, with $N \geq 2$
- The value of an impression to advertiser *i* is $X_i = CM_i$
- C is the (random) ***common value factor*** and
  - M_i is the (random) ***match value factor*** for bidder *i*

- **Key Assumptions**
  1. Advertiser 0 (the "brand advertiser') does not observe X_0
  2. Performance advertisers $n = 1, \dots, N$ observe their values X_n
     Define $X = (X_1, \dots, X_n)$.
  3. The common value factor C is statistically independent of the random vector $M \stackrel{\text{def}}{=} (M_0, \dots, M_N)$

A Market Design Approach

- Compare the "restricted-worst-case efficiency" (and later, revenues) of alternative mechanisms.
- The mechanisms considered are:

1. "Bayes optimal" mechanism
2. Our benchmark: "Omniscient" mechanism with C observed
3. Second-price auction
4. Our new "Modified second-bid auction" in which the highest performance bidder wins *if the ratio of the highest to second-highest performance bid exceeds a threshold.*

Bayesian Optimal Mechanism

OPT …and its drawbacks

Optimal Mechanism Formulation

- $z_i(X)$ is probability that i wins, given X
- $p_i(X)$ is i's expected payment, given X

- Efficiency Objective
  - Goal is to maximize $E[\sum_{i=0}^{n} X_i z_i(X)]$
    - subject to dominant-strategy incentive constraints and participation constraints
  - Let OPT be the mechanism that does that.

Example

- Assume that $M_1, \ldots, M_n$ are IID and that…

$$P\{C = 1\} = P\{C = 2\} = \frac{1}{2}$$

$$For\ j = 1,2,3, P\{M_j = 1\} = P\{M_j = 2\} = P\{M_j = 4\} = \frac{1}{3}$$

$$3 < E[M_0] < 4$$

- So, it is efficient to assign this impression to a performance advertiser $j \neq 0$ only if and only if $M_j = 4$.

OPT in the Example

- The expected-efficiency-maximizing assignment with $N = 2$ is:
  - There are two easy conditions to analyze:
    - If $X_{(1)} \in \{1,2\}$, then $M_{(1)} \leq 2 < E[M_0] \Rightarrow$ brand advertiser wins
    - If $X_{(1)} = 8$, then $M_{(1)} = 4 > E[M_0] \Rightarrow$ top performance advertiser wins
  - If $X_{(1)} = 4$, assignment hinges on $X_{(2)}$ and particularly whether $E[M_{(1)}|X_{(1)}, X_{(2)}] \gtrless E[M_0]$.
    - If $X_{(2)} = 1$, then $M_{(1)} = 4 \Rightarrow$ top performance advertiser wins
    - If $X_{(2)} = 2$ or 4, then $\mathrm{E}\left[\mathrm{M}_{(1)}\middle|X_{(1)}, X_{(2)}\right] = 3 < E[M_0] \Rightarrow$ brand advertiser wins
      - If $X_{(2)} = 2$, then $\Pr\left\{C = 1, M_{(1)} = 4, M_{(2)} = 2\middle|X_{(1)}, X_{(2)}\right\} = \Pr\left\{C = 2, M_{(1)} = 2, M_{(2)} = 1\middle|X_{(1)}, X_{(2)}\right\} = \frac{1}{2}$.
      - If $X_{(2)} = 4$, then $\Pr\left\{C = 1, M_{(1)} = M_{(2)} = 4\middle|X_{(1)}, X_{(2)}\right\} = \Pr\{C = 2, M_{(1)} = M_{(2)} = 2|X_{(1)}, X_{(2)}\} = \frac{1}{2}$.

Three Concerns about OPT

- The example highlights some troublesome attributes of OPT
  1. *Sensitivity:* OPT is sensitive to detailed distributional assumptions.
  2. *False-name bidding*: Performance advertiser n with value $X_n = 4$ can benefit by submitting a *additional*, false-name bid of $X_{\hat{n}} = 1$ (because that encourages the auctioneer to infer that $M_n = 4$ whenever X_n is the maximum performance value.)
  3. *Adverse selection*: The brand advertiser wins 4/9 of high-value impressions, but 7/9 of low-value ones.
     - This possibility can be problematic, especially if the brand advertiser is uninformed about the other bidders and the model parameters, and so is challenged even to estimate these fractions.

The Omniscient Benchmark

OMN, in which the auctioneer observes both the bids and C

OMN Benchmark

- Extreme assumption: the auctioneer can gather *perfect information* about the common factor C and can allocate without facing incentive constraints.
- Auctioneer could then achieve this value:

$$V(OMN) = E\left[\max(X_0, X_1, \ldots, X_n)\right],$$

$$where\ X_0 = CE[M_0]$$

- Performance of last two mechanisms is measured relative to $V(OMN)$.

MSB Characterization

Modified Second Bid auction characterized by its properties

Some Mechanism Properties

- A mechanism is
  - *anonymous* (*among performance advertisers*) if…
  - *strategy-proof* if…
  - *fully strategy-proof* if, in addition, it is both
    - *bidder false-name proof*: no bidder can benefit by submitting multiple bids, and
    - *publisher false-name proof*: the seller cannot benefit by submitting "low" bids (below all performance bids)
  - *adverse-selection free* if for every joint distribution on (C, M) such that C and M are independent, $z_0(X)$ is statistically independent of C.

Characterization Theorem

- **Definition.** A direct mechanism is a *modified second bid auction* if for some $\alpha \geq 1$,
  - If $\frac{X_{(1)}}{X_{(2)}} > \alpha$, then the highest performance advertiser wins & pays $\alpha X_{(2)}$.
  - If $\frac{X_{(1)}}{X_{(2)}} \leq \alpha$, then the brand advertiser wins (and pays its contract price).

- **Theorem.** A deterministic mechanism (z, p) is anonymous, fully strategy-proof, and adverse selection free *if and only if* it is a modified second bid auction.

Proof Ideas

1. Deterministic & strategy-proof mechanism ⇔ threshold auction.
2. …+Anonymous ⇔ the same threshold function for all performance bidders.
3. …+False-name proof ⇔ the threshold depends only on the second highest bid.
4. …+Adverse-selection free ⇔ the allocation depends on ratio of two highest bids.

Comparing MSB_{α} and SP_{r} to OMN

MSB_{α} : modified second-bid auction

SP_{r} : second-price auction with reserve

Assumptions for Comparison

- Evaluate MSB_α and SP_r mechanisms in worst case over a limited family of environments, in which…
  - $M_1, \dots, M_N$ are IID from a distribution F.
  - C is drawn from distribution G.
  - $N \geq 2$ and $E[M_0] \geq 0$ are free to vary.

Efficiency Performance

- **Theorem.** (Comparing SP_r and MSB_α to OMN)

1. Assuming Nash equilibrium bidding by the brand advertiser, both MSB and SP have similar worst case performance:

$$\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_{\alpha} \frac{V(MSB_\alpha)}{V(OMN)} = \frac{1}{2}$$

$$\inf_{F,G,N\geq 2,E[M_0]\geq 0} \max_{r} \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}$$

2. Further restricting F and/or G to be drawn from power law distributions $\mathcal{P}$,

$$\inf_{F\in\mathcal{P},G\in\mathcal{P},N\geq 2,E[M_0]\geq 0} \max_{r} \frac{V(SP_r)}{V(OMN)} = \frac{1}{2}$$

$$\inf_{F\in\mathcal{P},G,N\geq 2,E[M_0]\geq 0} \max_{\alpha} \frac{V(MSB_\alpha)}{V(OMN)} \approx 0.948$$

Revenue Performance

- **Theorem.** Fix a number of bidders N and assume that the publisher shares in the rents from brand advertising in any fixed proportions, say $(\delta, 1-\delta)$.
- If F is a power law distribution, then there is some α such that MSB_α achieves at least 94.8% of the expected revenue from the corresponding expected-revenue-maximizing strategy-proof auction $REVMAX$.

Conclusion

- Adverse selection can be neutralized, without encouraging false-name bidding, provided that $X_n = CM_n$ and C and M are independent.

- The cost of doing that is low, even without observing the common value factor C, provided that the tails of the distribution are fat (power law).

- For real applications, we need to evaluate…
  - Is adverse selection important?
  - Are match values independent?
  - Are match-value distributions fat-tailed?

End