

Robust LogitBoost and Adaptive Base Class (ABC) LogitBoost

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Abstract

Logitboost is an influential boosting algorithm for classification. In this paper, we develop **robust logitboost** to provide an explicit formulation of tree-split criterion for building weak learners (regression trees) for *logitboost*. This formulation leads to a numerically stable implementation of *logitboost*. We then propose **abc-logitboost** for multi-class classification, by combining *robust logitboost* with the prior work of *abc-boost*. Previously, *abc-boost* was implemented as *abc-mart* using the *mart* algorithm.

Our extensive experiments on multi-class classification compare four algorithms: *mart*, *abc-mart*, (*robust*) *logitboost*, and *abc-logitboost*, and demonstrate the superiority of *abc-logitboost*. Comparisons with other learning methods including SVM and **deep learning** are also available through prior publications.

1 Introduction

Boosting [14, 5, 6, 1, 15, 8, 13, 7, 4] has been successful in machine learning and industry practice. This study revisits *logitboost* [8], focusing on multi-class classification.

We denote a training dataset by $\{y_i, \mathbf{x}_i\}_{i=1}^N$, where N is the number of feature vectors (samples), \mathbf{x}_i is the i th feature vector, and $y_i \in \{0, 1, 2, \dots, K-1\}$ is the i th class label, where $K \geq 3$ in multi-class classification.

Both *logitboost* [8] and *mart* (multiple additive regression trees) [7] can be viewed as generalizations to the classical logistic regression, which models class probabilities $p_{i,k}$ as

$$p_{i,k} = \Pr(y_i = k | \mathbf{x}_i) = \frac{e^{F_{i,k}(\mathbf{x}_i)}}{\sum_{s=0}^{K-1} e^{F_{i,s}(\mathbf{x}_i)}}. \quad (1)$$

While logistic regression simply assumes $F_{i,k}(\mathbf{x}_i) = \beta_k^T \mathbf{x}_i$, *Logitboost* and *mart* adopt the flexible “additive model,” which is a function of M terms:

$$F^{(M)}(\mathbf{x}) = \sum_{m=1}^M \rho_m h(\mathbf{x}; \mathbf{a}_m), \quad (2)$$

where $h(\mathbf{x}; \mathbf{a}_m)$, the base (weak) learner, is typically a regression tree. The parameters, ρ_m and \mathbf{a}_m , are learned from the data, by maximum likelihood, which is equivalent to minimizing the *negative log-likelihood loss*

$$L = \sum_{i=1}^N L_i, \quad L_i = - \sum_{k=0}^{K-1} r_{i,k} \log p_{i,k} \quad (3)$$

where $r_{i,k} = 1$ if $y_i = k$ and $r_{i,k} = 0$ otherwise.

For identifiability, $\sum_{k=0}^{K-1} F_{i,k} = 0$, i.e., the **sum-to-zero** constraint, is usually adopted [8, 7, 17, 11, 16, 19, 18].

1.1 Logitboost

As described in Alg. 1, [8] builds the additive model (2) by a greedy stage-wise procedure, using a second-order (diagonal) approximation, which requires knowing the first two derivatives of the loss function (3) with respective to the function values $F_{i,k}$. [8] obtained:

$$\frac{\partial L_i}{\partial F_{i,k}} = -(r_{i,k} - p_{i,k}), \quad \frac{\partial^2 L_i}{\partial F_{i,k}^2} = p_{i,k}(1 - p_{i,k}). \quad (4)$$

While [8] assumed the sum-to-zero constraint, they showed (4) by conditioning on a “base class” and noticed the resultant derivatives were independent of the choice of the base.

Algorithm 1 Logitboost [8, Alg. 6]. ν is the shrinkage.

- 0: $r_{i,k} = 1$, if $y_i = k$, $r_{i,k} = 0$ otherwise.
- 1: $F_{i,k} = 0$, $p_{i,k} = \frac{1}{K}$, $k = 0$ to $K-1$, $i = 1$ to N
- 2: For $m = 1$ to M Do
- 3: For $k = 0$ to $K-1$, Do
- 4: Compute $w_{i,k} = p_{i,k}(1 - p_{i,k})$.
- 5: Compute $z_{i,k} = \frac{r_{i,k} - p_{i,k}}{p_{i,k}(1 - p_{i,k})}$.
- 6: Fit the function $f_{i,k}$ by a weighted least-square of $z_{i,k}$
- 7: to \mathbf{x}_i with weights $w_{i,k}$.
- 8: End
- 9: $F_{i,k} = F_{i,k} + \nu \frac{K-1}{K} \left(f_{i,k} - \frac{1}{K} \sum_{k=0}^{K-1} f_{i,k} \right)$
- 10: End

At each stage, *logitboost* fits an individual regression function separately for each class. This diagonal approximation appears to be a must if the base learner is implemented using regression trees. For industry applications, using trees as the weak learner appears to be the standard practice.

1.2 Adaptive Base Class Boost

[12] derived the derivatives of (3) under the sum-to-zero constraint. Without loss of generality, we can assume that class 0 is the base class. For any $k \neq 0$,

$$\frac{\partial L_i}{\partial F_{i,k}} = (r_{i,0} - p_{i,0}) - (r_{i,k} - p_{i,k}), \quad (5)$$

$$\frac{\partial^2 L_i}{\partial F_{i,k}^2} = p_{i,0}(1 - p_{i,0}) + p_{i,k}(1 - p_{i,k}) + 2p_{i,0}p_{i,k}. \quad (6)$$

The base class must be identified at each boosting iteration during training. [12] suggested an exhaustive procedure to adaptively find the best base class to minimize the training loss (3) at each iteration. [12] combined the idea of *abc-boost* with *mart*, to develop *abc-mart*, which achieved good performance in multi-class classification.

It was believed that *logitboost* could be numerically unstable [8, 7, 9, 3]. In this paper, we provide an explicit formulation for tree construction to demonstrate that *logitboost* is actually stable. We name this construction **robust logitboost**. We then combine the idea of *robust logitboost* with *abc-boost* to develop **abc-logitboost**, for multi-class classification, which often considerably improves *abc-mart*.

2 Robust Logitboost

In practice, tree is the default weak learner. The next subsection presents the tree-split criterion of *robust logitboost*.

2.1 Tree-Split Criterion Using 2nd-order Information

Consider N weights w_i , and N response values z_i , $i = 1$ to N , which are assumed to be ordered according to the sorted order of the corresponding feature values. The tree-split procedure is to find the index s , $1 \leq s < N$, such that the weighted square error (SE) is reduced the most if split at s . That is, we seek the s to maximize

$$Gain(s) = SE_T - (SEL + SER) \\ = \sum_{i=1}^N (z_i - \bar{z})^2 w_i - \left[\sum_{i=1}^s (z_i - \bar{z}_L)^2 w_i + \sum_{i=s+1}^N (z_i - \bar{z}_R)^2 w_i \right]$$

where

$$\bar{z} = \frac{\sum_{i=1}^N z_i w_i}{\sum_{i=1}^N w_i}, \quad \bar{z}_L = \frac{\sum_{i=1}^s z_i w_i}{\sum_{i=1}^s w_i}, \quad \bar{z}_R = \frac{\sum_{i=s+1}^N z_i w_i}{\sum_{i=s+1}^N w_i}.$$

We can simplify the expression for $Gain(s)$ to be:

$$Gain(s) = \sum_{i=1}^N (z_i^2 + \bar{z}^2 - 2\bar{z}z_i) w_i \\ - \sum_{i=1}^s (z_i^2 + \bar{z}_L^2 - 2\bar{z}_L z_i) w_i - \sum_{i=s+1}^N (z_i^2 + \bar{z}_R^2 - 2\bar{z}_R z_i) w_i \\ = \sum_{i=1}^N (\bar{z}^2 - 2\bar{z}z_i) w_i - \sum_{i=1}^s (\bar{z}_L^2 - 2\bar{z}_L z_i) w_i - \sum_{i=s+1}^N (\bar{z}_R^2 - 2\bar{z}_R z_i) w_i \\ = \left[\bar{z}^2 \sum_{i=1}^N w_i - 2\bar{z} \sum_{i=1}^N z_i w_i \right] - \left[\bar{z}_L^2 \sum_{i=1}^s w_i - 2\bar{z}_L \sum_{i=1}^s z_i w_i \right] - \left[\bar{z}_R^2 \sum_{i=s+1}^N w_i - 2\bar{z}_R \sum_{i=s+1}^N z_i w_i \right]$$

$$Gain(s) = \left[-\bar{z} \sum_{i=1}^N z_i w_i \right] - \left[-\bar{z}_L \sum_{i=1}^s z_i w_i \right] - \left[-\bar{z}_R \sum_{i=s+1}^N z_i w_i \right] \\ = \frac{[\sum_{i=1}^s z_i w_i]^2}{\sum_{i=1}^s w_i} + \frac{[\sum_{i=s+1}^N z_i w_i]^2}{\sum_{i=s+1}^N w_i} - \frac{[\sum_{i=1}^N z_i w_i]^2}{\sum_{i=1}^N w_i}$$

Plugging in $w_i = p_{i,k}(1 - p_{i,k})$, $z_i = \frac{r_{i,k} - p_{i,k}}{p_{i,k}(1 - p_{i,k})}$ yields

$$Gain(s) = \frac{[\sum_{i=1}^s (r_{i,k} - p_{i,k})]^2}{\sum_{i=1}^s p_{i,k}(1 - p_{i,k})} + \frac{[\sum_{i=s+1}^N (r_{i,k} - p_{i,k})]^2}{\sum_{i=s+1}^N p_{i,k}(1 - p_{i,k})} \\ - \frac{[\sum_{i=1}^N (r_{i,k} - p_{i,k})]^2}{\sum_{i=1}^N p_{i,k}(1 - p_{i,k})}. \quad (7)$$

There are at least two ways to see why the criterion given by (7) is numerically stable. First of all, the computations involve $\sum p_{i,k}(1 - p_{i,k})$ as a group. It is much less likely that $p_{i,k}(1 - p_{i,k}) \approx 0$ for all i 's in the region. Secondly, if indeed that $p_{i,k}(1 - p_{i,k}) \rightarrow 0$ for all i 's in this region, it means the model is fitted perfectly, i.e., $p_{i,k} \rightarrow r_{i,k}$. In other words, (e.g.,) $[\sum_{i=1}^N (r_{i,k} - p_{i,k})]^2$ in (7) also approaches zero at the square rate.

2.2 The Robust Logitboost Algorithm

Algorithm 2 *Robust logitboost*, which is very similar to Friedman's *mart* algorithm [7], except for Line 4.

- 1: $F_{i,k} = 0$, $p_{i,k} = \frac{1}{K}$, $k = 0$ to $K - 1$, $i = 1$ to N
 - 2: For $m = 1$ to M Do
 - 3: For $k = 0$ to $K - 1$ Do
 - 4: $\{R_{j,k,m}\}_{j=1}^J$ = J -terminal node regression tree from $\{r_{i,k} - p_{i,k}, \mathbf{x}_i\}_{i=1}^N$, with weights $p_{i,k}(1 - p_{i,k})$ as in (7)
 - 5: $\beta_{j,k,m} = \frac{K-1}{K} \frac{\sum_{\mathbf{x}_i \in R_{j,k,m}} r_{i,k} - p_{i,k}}{\sum_{\mathbf{x}_i \in R_{j,k,m}} (1 - p_{i,k}) p_{i,k}}$
 - 6: $F_{i,k} = F_{i,k} + \nu \sum_{j=1}^J \beta_{j,k,m} \mathbf{1}_{\mathbf{x}_i \in R_{j,k,m}}$
 - 7: End
 - 8: $p_{i,k} = \exp(F_{i,k}) / \sum_{s=0}^{K-1} \exp(F_{i,s})$
 - 9: End
-

Alg. 2 describes *robust logitboost* using the tree-split criterion (7). Note that after trees are constructed, the values of the terminal nodes are computed by

$$\frac{\sum_{node} z_{i,k} w_{i,k}}{\sum_{node} w_{i,k}} = \frac{\sum_{node} r_{i,k} - p_{i,k}}{\sum_{node} p_{i,k}(1 - p_{i,k})}, \quad (8)$$

which explains Line 5 of Alg. 2.

2.3 Friedman's Mart Algorithm

Friedman [7] proposed *mart* (*multiple additive regression trees*), a creative combination of gradient descent and Newton's method, by using the first-order information to construct the trees and using both the first- & second-order information to determine the values of the terminal nodes.

Corresponding to (7), the tree-split criterion of *mart* is

$$\begin{aligned} MartGain(s) = & \frac{1}{s} \left[\sum_{i=1}^s (r_{i,k} - p_{i,k}) \right]^2 \\ & + \frac{1}{N-s} \left[\sum_{i=s+1}^N (r_{i,k} - p_{i,k}) \right]^2 - \frac{1}{N} \left[\sum_{i=1}^N (r_{i,k} - p_{i,k}) \right]^2. \end{aligned} \quad (9)$$

In Sec. 2.1, plugging in responses $z_{i,k} = r_{i,k} - p_{i,k}$ and weights $w_i = 1$, yields (9).

Once the tree is constructed, Friedman [7] applied a one-step Newton update to obtain the values of the terminal nodes. Interestingly, this one-step Newton update yields exactly the same equation as (8). In other words, (8) is interpreted as weighted average in *logitboost* but it is interpreted as the one-step Newton update in *mart*.

Therefore, the *mart* algorithm is similar to Alg. 2; we only need to change Line 4, by replacing (7) with (9).

In fact, Eq. (8) also provides one more explanation why the tree-split criterion (7) is numerically stable, because (7) is always numerically more stable than (8). The update formula (8) has been successfully used in practice for 10 years since the advent of *mart*.

2.4 Experiments on Binary Classification

While we focus on multi-class classification, we also provide some experiments on binary classification in App. A.

3 Adaptive Base Class (ABC) Logitboost

Developed by [12], the *abc-boost* algorithm consists of the following two components:

1. Using the widely-used *sum-to-zero* constraint [8, 7, 17, 11, 16, 19, 18] on the loss function, one can formulate boosting algorithms only for $K-1$ classes, by using one class as the **base class**.
2. At each boosting iteration, **adaptively** select the base class according to the training loss (3). [12] suggested an exhaustive search strategy.

Abc-boost by itself is not a concrete algorithm. [12] developed *abc-mart* by combining *abc-boost* with *mart*. In this paper, we develop **abc-logitboost**, a new algorithm by combining *abc-boost* with (*robust*) *logitboost*.

Alg. 3 presents *abc-logitboost*, using the derivatives in (5) and (6) and the same exhaustive search strategy as used by *abc-mart*. Again, *abc-logitboost* differs from *abc-mart* only in the tree-split procedure (Line 5 in Alg. 3).

Compared to Alg. 2, *abc-logitboost* differs from (*robust*) *logitboost* in that they use different derivatives and *abc-logitboost* needs an additional loop to select the base class at each boosting iteration.

Algorithm 3 *Abc-logitboost* using the exhaustive search strategy for the base class, as suggested in [12]. The vector B stores the base class numbers.

```

1:  $F_{i,k} = 0$ ,  $p_{i,k} = \frac{1}{K}$ ,  $k = 0$  to  $K-1$ ,  $i = 1$  to  $N$ 
2: For  $m = 1$  to  $M$  Do
3:   For  $b = 0$  to  $K-1$ , Do
4:     For  $k = 0$  to  $K-1$ ,  $k \neq b$ , Do
5:        $\{R_{j,k,m}\}_{j=1}^J$  =  $J$ -terminal node regression tree from
:        $\{-(r_{i,b} - p_{i,b}) + (r_{i,k} - p_{i,k}), \mathbf{x}_i\}_{i=1}^N$  with weights
:        $p_{i,b}(1-p_{i,b}) + p_{i,k}(1-p_{i,k}) + 2p_{i,b}p_{i,k}$ , as in Sec. 2.1.
6:        $\beta_{j,k,m} = \frac{\sum_{\mathbf{x}_i \in R_{j,k,m}} -(r_{i,b} - p_{i,b}) + (r_{i,k} - p_{i,k})}{\sum_{\mathbf{x}_i \in R_{j,k,m}} p_{i,b}(1-p_{i,b}) + p_{i,k}(1-p_{i,k}) + 2p_{i,b}p_{i,k}}$ 
7:        $G_{i,k,b} = F_{i,k} + \nu \sum_{j=1}^J \beta_{j,k,m} \mathbf{1}_{\mathbf{x}_i \in R_{j,k,m}}$ 
8:     End
9:    $G_{i,b,b} = -\sum_{k \neq b} G_{i,k,b}$ 
10:   $q_{i,k} = \exp(G_{i,k,b}) / \sum_{s=0}^{K-1} \exp(G_{i,s,b})$ 
11:   $L^{(b)} = -\sum_{i=1}^N \sum_{k=0}^{K-1} r_{i,k} \log(q_{i,k})$ 
12: End
13:  $B(m) = \operatorname{argmin}_b L^{(b)}$ 
14:  $F_{i,k} = G_{i,k,B(m)}$ 
15:  $p_{i,k} = \exp(F_{i,k}) / \sum_{s=0}^{K-1} \exp(F_{i,s})$ 
16: End

```

3.1 Why Does the Choice of Base Class Matter?

It matters because of the diagonal approximation; that is, fitting a regression tree for each class at each boosting iteration. To see this, we can take a look at the Hessian matrix, for $K = 3$. Using the original logitboost/mart derivatives (4), the determinant of the Hessian matrix is

$$= \begin{vmatrix} \frac{\partial^2 L_i}{\partial p_0^2} & \frac{\partial^2 L_i}{\partial p_0 \partial p_1} & \frac{\partial^2 L_i}{\partial p_0 \partial p_2} \\ \frac{\partial^2 L_i}{\partial p_1 \partial p_0} & \frac{\partial^2 L_i}{\partial p_1^2} & \frac{\partial^2 L_i}{\partial p_1 \partial p_2} \\ \frac{\partial^2 L_i}{\partial p_2 \partial p_0} & \frac{\partial^2 L_i}{\partial p_2 \partial p_1} & \frac{\partial^2 L_i}{\partial p_2^2} \end{vmatrix} = \begin{vmatrix} p_0(1-p_0) & -p_0p_1 & -p_0p_2 \\ -p_1p_0 & p_1(1-p_1) & -p_1p_2 \\ -p_2p_0 & -p_2p_1 & p_2(1-p_2) \end{vmatrix} = 0$$

as expected, because there are only $K-1$ degrees of freedom. A simple fix is to use the diagonal approximation [8, 7]. In fact, when trees are used as the base learner, it seems one must use the diagonal approximation.

Next, we consider the derivatives (5) and (6). This time, when $K = 3$ and $k = 0$ is the base class, we only have a 2 by 2 Hessian matrix, whose determinant is

$$\begin{vmatrix} \frac{\partial^2 L_i}{\partial p_1^2} & \frac{\partial^2 L_i}{\partial p_1 \partial p_2} \\ \frac{\partial^2 L_i}{\partial p_2 \partial p_1} & \frac{\partial^2 L_i}{\partial p_2^2} \end{vmatrix} = \begin{vmatrix} p_0(1-p_0) + p_1(1-p_1) + 2p_0p_1 & p_0 - p_0^2 + p_0p_1 + p_0p_2 - p_1p_2 \\ p_0 - p_0^2 + p_0p_1 + p_0p_2 - p_1p_2 & p_0(1-p_0) + p_2(1-p_2) + 2p_0p_2 \end{vmatrix} = \begin{vmatrix} p_0p_1 + p_0p_2 + p_1p_2 - p_0p_1^2 - p_0p_2^2 - p_1p_2^2 - p_2p_1^2 - p_1p_0^2 - p_2p_0^2 + 6p_0p_1p_2, \end{vmatrix}$$

which is non-zero and is in fact independent of the choice of the base class (even though we assume $k = 0$ as the base

in this example). In other words, the choice of the base class would not matter if the full Hessian is used.

However, the choice of the base class will matter because we will have to use diagonal approximation in order to construct trees at each iteration.

4 Experiments on Multi-class Classification

4.1 Datasets

Table 1 lists the datasets used in our study.

Table 1: Datasets

dataset	K	# training	# test	# features
Covertype290k	7	290506	290506	54
Covertype145k	7	145253	290506	54
Poker25k	10	525010	500000	25
Poker275k	10	275010	500000	25
Poker150k	10	150010	500000	25
Poker100k	10	100010	500000	25
Poker25kT1	10	25010	500000	25
Poker25kT2	10	25010	500000	25
Mnist10k	10	10000	60000	784
M-Basic	10	12000	50000	784
M-Rotate	10	12000	50000	784
M-Image	10	12000	50000	784
M-Rand	10	12000	50000	784
M-RotImg	10	12000	50000	784
M-Noise1	10	10000	2000	784
M-Noise2	10	10000	2000	784
M-Noise3	10	10000	2000	784
M-Noise4	10	10000	2000	784
M-Noise5	10	10000	2000	784
M-Noise6	10	10000	2000	784
Letter15k	26	15000	5000	16
Letter4k	26	4000	16000	16
Letter2k	26	2000	18000	16

Covertype The original UCI *Covertype* dataset is fairly large, with 581012 samples. To generate *Covertype290k*, we randomly split the original data into halves, one half for training and another half for testing. For *Covertype145k*, we randomly select one half from the training set of *Covertype290k* and still keep the same test set.

Poker The UCI *Poker* dataset originally had 25010 samples for training and 1000000 samples for testing. Since the test set is very large, we randomly divide it equally into two parts (I and II). *Poker25kT1* uses the original training set for training and Part I of the original test set for testing. *Poker25kT2* uses the original training set for training and Part II of the original test set for testing. This way, *Poker25kT1* can use the test set of *Poker25kT2* for validation, and *Poker25kT2* can use the test set of *Poker25kT1* for validation. The two test sets are still very large.

In addition, we enlarge the training set to form *Poker525k*, *Poker275k*, *Poker150k*, *Poker100k*. All four enlarged training sets use the same test set as *Poker25kT2* (i.e., Part II of the original test set). The training set of *Poker525k* contains the original (25k) training set plus Part I of the original test set. The training set of *Poker275k/Poker150k/Poker100k* contains the original training set plus 250k/125k/75k samples from Part I of the original test set.

Mnist While the original *Mnist* dataset is extremely popular, it is known to be too easy [10]. Originally, *Mnist*

used 60000 samples for training and 10000 samples for testing. *Mnist10k* uses the original (10000) test set for training and the original (60000) training set for testing.

Mnist with Many Variations

[10] created a variety of difficult datasets by adding background (correlated) noises, background images, rotations, etc, to the original *Mnist* data. We shortened the names of the datasets to be *M-Basic*, *M-Rotate*, *M-Image*, *M-Rand*, *M-RotImg*, and *M-Noise1*, *M-Noise2* to *M-Noise6*.

Letter The UCI *Letter* dataset has in total 20000 samples. In our experiments, *Letter4k* (*Letter2k*) use the last 4000 (2000) samples for training and the rest for testing. The purpose is to demonstrate the performance of the algorithms using only small training sets. We also include *Letter15k*, which is one of the standard partitions, by using 15000 samples for training and 5000 samples for testing.

4.2 The Main Goal of Our Experiments

The main goal of our experiments is to demonstrate that

1. *Abc-logitboost* and *abc-mart* outperform (*robust*) *logitboost* and *mart*, respectively.
2. (*Robust*) *logitboost* often outperforms *mart*.
3. *Abc-logitboost* often outperforms *abc-mart*.
4. The improvements hold for (almost) all reasonable parameters, not just for a few selected sets of parameters.

The main parameter is J , the number of terminal tree nodes. It is often the case that test errors are not very sensitive to the shrinkage parameter ν , provided $\nu \leq 0.1$ [7, 3].

4.3 Detailed Experiment Results on *Mnist10k*, *M-Image*, *Letter4k*, and *Letter2k*

For these datasets, we experiment with every combination of $J \in \{4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 30, 40, 50\}$ and $\nu \in \{0.04, 0.06, 0.08, 0.1\}$. We train the four boosting algorithms till the training loss (3) is close to the machine accuracy to exhaust the capacity of the learners, for reliable comparisons, up to $M = 10000$ iterations. We report the test mis-classification errors at the last iterations.

For *Mnist10k*, Table 2 presents the test mis-classification errors, which verifies the consistent improvements of (A) *abc-logitboost* over (*robust*) *logitboost*, (B) *abc-logitboost* over *abc-mart*, (C) (*robust*) *logitboost* over *mart*, and (D) *abc-mart* over *mart*. The table also verifies that the performances are not too sensitive to the parameters, especially considering the number of test samples is 60000. In App. B, Table 12 reports the testing P -values for every combination of J and ν .

Table 3, 4 , 5 present the test mis-classification errors on *M-Image*, *Letter4k*, and *Letter2k*, respectively.

Fig. 1 provides the test errors for all boosting iterations. While we believe this is the most reliable comparison, unfortunately there is no space to present them all.

Table 2: **Mnist10k**. Upper table: The test mis-classification errors of *mart* and *abc-mart* (bold numbers). Bottom table: The test errors of *logitboost* and *abc-logitboost* (bold numbers)

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	3356 3060	3329 3019	3318 2855	3326 2794
$J = 6$	3185 2760	3093 2626	3129 2656	3217 2590
$J = 8$	3049 2558	3054 2555	3054 2534	3035 2577
$J = 10$	3020 2547	2973 2521	2990 2520	2978 2506
$J = 12$	2927 2498	2917 2457	2945 2488	2907 2490
$J = 14$	2925 2487	2901 2471	2877 2470	2884 2454
$J = 16$	2899 2478	2893 2452	2873 2465	2860 2451
$J = 18$	2857 2469	2880 2460	2870 2437	2855 2454
$J = 20$	2833 2441	2834 2448	2834 2444	2815 2440
$J = 24$	2840 2447	2827 2431	2801 2427	2784 2455
$J = 30$	2826 2457	2822 2443	2828 2470	2807 2450
$J = 40$	2837 2482	2809 2440	2836 2447	2782 2506
$J = 50$	2813 2502	2826 2459	2824 2469	2786 2499
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	2936 2630	2970 2600	2980 2535	3017 2522
$J = 6$	2710 2263	2693 2252	2710 2226	2711 2223
$J = 8$	2599 2159	2619 2138	2589 2120	2597 2143
$J = 10$	2553 2122	2527 2118	2516 2091	2500 2097
$J = 12$	2472 2084	2468 2090	2464 2095	
$J = 14$	2451 2083	2420 2094	2432 2063	2419 2050
$J = 16$	2424 2111	2437 2114	2393 2097	2395 2082
$J = 18$	2399 2088	2402 2087	2389 2088	2380 2097
$J = 20$	2388 2128	2414 2112	2411 2095	2381 2102
$J = 24$	2442 2174	2415 2147	2417 2129	2419 2138
$J = 30$	2468 2235	2434 2237	2423 2221	2449 2177
$J = 40$	2551 2310	2509 2284	2518 2257	2531 2260
$J = 50$	2612 2353	2622 2359	2579 2332	2570 2341

Table 4: **Letter4k**. Upper table: The test mis-classification errors of *mart* and *abc-mart* (bold numbers). Bottom table: The test errors of *logitboost* and *abc-logitboost* (bold numbers)

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	1681 1415	1660 1380	1671 1368	1655 1323
$J = 6$	1618 1320	1584 1288	1588 1266	1577 1240
$J = 8$	1531 1266	1522 1246	1516 1192	1521 1184
$J = 10$	1499 1228	1463 1208	1479 1186	1470 1185
$J = 12$	1420 1213	1434 1186	1409 1170	1437 1162
$J = 14$	1410 1190	1388 1156	1377 1151	1396 1160
$J = 16$	1395 1167	1402 1156	1396 1157	1387 1146
$J = 18$	1376 1164	1375 1139	1357 1127	1352 1152
$J = 20$	1386 1154	1397 1130	1371 1131	1370 1149
$J = 24$	1371 1148	1348 1155	1374 1164	1391 1150
$J = 30$	1383 1174	1406 1174	1401 1177	1404 1209
$J = 40$	1458 1211	1455 1224	1441 1233	1454 1215
$J = 50$	1484 1203	1517 1233	1487 1248	1522 1250
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	1460 1296	1471 1241	1452 1202	1446 1208
$J = 6$	1390 1143	1394 1117	1382 1090	1374 1074
$J = 8$	1336 1089	1332 1080	1311 1066	1297 1046
$J = 10$	1289 1062	1285 1067	1380 1034	1273 1049
$J = 12$	1251 1058	1247 1069	1261 1044	1243 1051
$J = 14$	1247 1063	1233 1051	1251 1040	1244 1066
$J = 16$	1244 1074	1227 1068	1231 1047	1228 1046
$J = 18$	1243 1059	1250 1040	1234 1052	1220 1057
$J = 20$	1226 1084	1242 1070	1242 1058	1235 1055
$J = 24$	1245 1079	1234 1059	1235 1058	1215 1073
$J = 30$	1232 1057	1247 1085	1229 1069	1230 1065
$J = 40$	1246 1095	1255 1093	1230 1094	1231 1087
$J = 50$	1248 1100	1230 1108	1233 1120	1246 1136

Table 3: **M-Image**. Upper table: The test mis-classification errors of *mart* and *abc-mart* (bold numbers). Bottom table: The test of *logitboost* and *abc-logitboost* (bold numbers)

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	6536 5867	6511 5813	6496 5774	6449 5756
$J = 6$	6203 5471	6174 5414	6176 5394	6139 5370
$J = 8$	6095 5320	6081 5251	6132 5141	6220 5181
$J = 10$	6076 5138	6104 5100	6154 5086	5332 4983
$J = 12$	6036 4963	6086 4956	6104 4926	6117 4867
$J = 14$	5922 4885	6037 4866	6018 4789	5993 4839
$J = 16$	5914 4847	5937 4806	5940 4797	5883 4766
$J = 18$	5955 4835	5886 4778	5896 4733	5814 4730
$J = 20$	5870 4749	5847 4722	5829 4707	5821 4727
$J = 24$	5816 4725	5766 4659	5785 4662	5752 4625
$J = 30$	5729 4649	5738 4629	5724 4626	5702 4654
$J = 40$	5752 4619	5699 4636	5672 4597	5676 4660
$J = 50$	5760 4674	5731 4667	5723 4659	5725 4649
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	5837 5539	5852 5480	5834 5408	5802 5430
$J = 6$	5473 5076	5471 4925	5457 4950	5437 4919
$J = 8$	5294 4756	5285 4748	5193 4678	5187 4670
$J = 10$	5141 4597	5120 4572	5052 4524	5049 4537
$J = 12$	5013 4432	5016 4455	4987 4416	4961 4389
$J = 14$	4914 4378	4922 4338	4906 4356	4895 4299
$J = 16$	4863 4317	4842 4307	4816 4279	4806 4314
$J = 18$	4762 4301	4740 4255	4754 4230	4751 4287
$J = 20$	4714 4251	4734 4231	4693 4214	4703 4268
$J = 24$	4676 4242	4610 4298	4663 4226	4638 4250
$J = 30$	4653 4351	4662 4307	4633 4311	4643 4286
$J = 40$	4713 4434	4724 4426	4760 4439	4768 4388
$J = 50$	4763 4502	4795 4534	4792 4487	4799 4479

Table 5: **Letter2k**. Upper table: The test mis-classification errors of *mart* and *abc-mart* (bold numbers). Bottom table: The test errors of *logitboost* and *abc-logitboost* (bold numbers)

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	2694 2512	2698 2470	2684 2419	2689 2435
$J = 6$	2683 2360	2664 2321	2640 2313	2629 2321
$J = 8$	2569 2279	2603 2289	2563 2259	2571 2251
$J = 10$	2534 2242	2516 2215	2504 2210	2491 2185
$J = 12$	2503 2202	2516 2215	2473 2198	2492 2201
$J = 14$	2488 2203	2467 2231	2460 2204	2460 2183
$J = 16$	2503 2219	2501 2219	2496 2235	2500 2205
$J = 18$	2494 2225	2497 2212	2472 2205	2439 2213
$J = 20$	2499 2199	2512 2198	2504 2188	2482 2220
$J = 24$	2549 2200	2549 2191	2526 2218	2538 2248
$J = 30$	2579 2237	2566 2232	2574 2244	2574 2285
$J = 40$	2641 2303	2632 2304	2606 2271	2667 2351
$J = 50$	2668 2382	2670 2362	2638 2413	2717 2367
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	2629 2347	2582 2299	2580 2256	2572 2231
$J = 6$	2427 2136	2450 2120	2428 2072	2429 2077
$J = 8$	2336 2080	2321 2049	2326 2035	2313 2037
$J = 10$	2316 2044	2306 2003	2314 2021	2307 2002
$J = 12$	2315 2024	2315 1992	2333 2018	2290 2018
$J = 14$	2317 2022	2305 2004	2315 2006	2292 2030
$J = 16$				

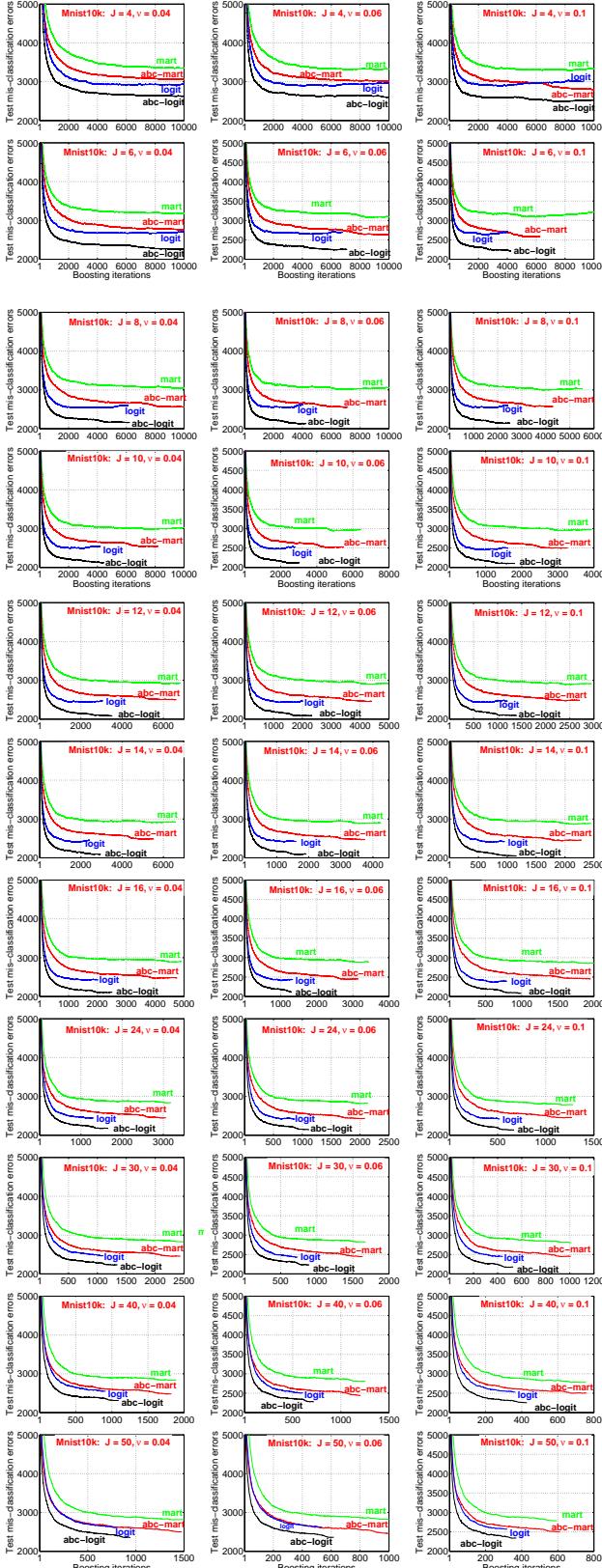


Figure 1: **Mnist10k**. Test mis-classification errors of four boosting algorithms, for shrinkage $\nu = 0.04$ (left), 0.06 (middle), 0.1 (right), and selected J terminal nodes.

4.4 Experiment Results on *Poker25kT1*, *Poker25kT2*

Recall, to provide a reliable comparison (and validation), we form two datasets *Poker25kT1* and *Poker25kT2* by equally dividing the original test set (1000000 samples) into two parts (I and II). Both use the same training set. *Poker25kT1* uses Part I of the original test set for testing and *Poker25kT2* uses Part II for testing.

Table 6 and Table 7 present the test mis-classification errors, for $J \in \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$, $\nu \in \{0.04, 0.06, 0.08, 0.1\}$, and $M = 10000$ boosting iterations (the machine accuracy is not reached). Comparing these two tables, we can see the corresponding entries are very close to each other, which again verifies that the four boosting algorithms provide reliable results on this dataset. Unlike *Mnist10k*, the test errors, especially using *mart* and *logitboost*, are slightly sensitive to the parameter J .

Table 6: **Poker25kT1**. Upper table: The test mis-classification errors of *mart* and *abc-mart* (bold numbers). Bottom table: The test of *logitboost* and *abc-logitboost* (bold numbers).

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	145880 90323	132526 67417	124283 49403	113985 42126
$J = 6$	71628 38017	59046 36839	48064 35467	43573 34879
$J = 8$	64090 39220	53400 37112	47360 36407	44131 35777
$J = 10$	60456 39661	52464 38547	47203 36990	46351 36647
$J = 12$	61452 41362	52697 39221	46822 37723	46965 37345
$J = 14$	58348 42764	56047 40993	50476 40155	47935 37780
$J = 16$	63518 44386	55418 43360	50612 41952	49179 40050
$J = 18$	64426 46463	55708 45607	54033 45838	52113 43040
$J = 20$	65528 49577	59236 47901	56384 45725	53506 44295
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	147064 102905	140068 71450	128161 51226	117085 42140
$J = 6$	81566 43156	59324 39164	51526 37954	48516 37546
$J = 8$	68278 46076	56922 40162	52532 38422	46789 37345
$J = 10$	63796 44830	55834 40754	53262 40486	47118 38141
$J = 12$	66732 48412	56867 44886	51248 42100	47485 39798
$J = 14$	64263 52479	55614 48093	51735 44688	47806 43048
$J = 16$	67092 53363	58019 51308	53746 47831	51267 46968
$J = 18$	69104 57147	56514 55468	55290 50292	51871 47986
$J = 20$	68899 62345	61314 57677	56648 53696	51608 49864

Table 7: **Poker25kT2**. The test mis-classification errors.

	<i>mart</i>	<i>abc-mart</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	144020 89608	131243 67071	123031 48855	113232 41688
$J = 6$	71004 37567	58487 36345	47564 34920	42935 34326
$J = 8$	63452 38703	52990 36586	46914 35836	43647 35129
$J = 10$	60061 39078	52125 38025	46912 36455	45863 36076
$J = 12$	61098 40834	52296 38657	46458 37203	46698 36781
$J = 14$	57924 42348	55622 40363	50243 39613	47619 37243
$J = 16$	63213 44067	55206 42973	50322 41485	48966 39446
$J = 18$	64056 46050	55461 45133	53652 45308	51870 42485
$J = 20$	65215 49046	58911 47430	56009 45390	53213 43888
	<i>logitboost</i>	<i>abc-logit</i>		
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$	$\nu = 0.1$
$J = 4$	145368 102014	138734 70886	126980 50783	116346 41551
$J = 6$	80782 42699	58769 38592	51202 37397	48199 36914
$J = 8$	68065 45737	56678 39648	52504 37935	46600 36731
$J = 10$	63153 44517	55419 40286	52835 40044	46913 37504
$J = 12$	66240 47948	56619 44602	50918 41582	47128 39378
$J = 14$	63763 52063	55238 47642	51526 44296	47545 42720
$J = 16$	66543 52937	57473 50842	53287 47578	51106 46635
$J = 18$	68477 56803	57070 55166	54954 49956	51603 47707
$J = 20$	68311 61980	61047 57383	56474 53364	51242 49506

4.5 Summary of Test Mis-classification Errors

Table 8 summarizes the test mis-classification errors. Since the test errors are not too sensitive to the parameters, for all datasets except *Poker25kT1* and *Poker25kT2*, we simply report the test errors with tree size $J = 20$ and shrinkage $\nu = 0.1$. (More tuning will possibly improve the results.)

For *Poker25kT1* and *Poker25kT2*, as we notice the performance is somewhat sensitive to the parameters, we use each others' test set as the validation set to report the test errors.

For *Covertype290k*, *Poker525k*, *Poker275k*, *Poker150k*, and *Poker100k*, as they are fairly large, we only train $M = 5000$ boosting iterations. For all other datasets, we always train $M = 10000$ iterations or terminate when the training loss (3) is close to the machine accuracy. Since we do not notice obvious over-fitting on these datasets, we simply report the test errors at the last iterations.

Table 8 also includes the results of regular logistic regression. It is interesting that the test errors are all the same (248892) for *Poker525k*, *Poker275k*, *Poker150k*, and *Poker100k* (but the predicted probabilities are different).

Table 8: Summary of test mis-classification errors.

Dataset	mart	abc-mart	logit	abc-logit	logi. regres.
Covertype290k	11350	10454	10765	9727	80233
Covertype145k	15767	14665	14928	13986	80314
Poker525k	7061	2424	2704	1736	248892
Poker275k	15404	3679	6533	2727	248892
Poker150k	22289	12340	16163	5104	248892
Poker100k	27871	21293	25715	13707	248892
Poker25kT1	43573	34879	46789	37345	250110
Poker25kT2	42935	34326	46600	36731	249056
Mnist10k	2815	2440	2381	2102	13950
M-Basic	2058	1843	1723	1602	10993
M-Rotate	7674	6634	6813	5959	26584
M-Image	5821	4727	4703	4268	19353
M-Rand	6577	5300	5020	4725	18189
M-RotImg	24912	23072	22962	22343	33216
M-Noise1	305	245	267	234	935
M-Noise2	325	262	270	237	940
M-Noise3	310	264	277	238	954
M-Noise4	308	243	256	238	933
M-Noise5	294	249	242	227	867
M-Noise6	279	224	226	201	788
Letter15k	155	125	139	109	1130
Letter4k	1370	1149	1252	1055	3712
Letter2k	2482	2220	2309	2034	4381

P-values

Table 9 summarizes four types of *P*-values:

- *P1*: for testing if *abc-mart* has significantly lower **error rates** than *mart*.
- *P2*: for testing if (*robust*) *logitboost* has significantly lower error rates than *mart*.
- *P3*: for testing if *abc-logitboost* has significantly lower error rates than *abc-mart*.
- *P4*: for testing if *abc-logitboost* has significantly lower error rates than (*robust*) *logitboost*.

The *P*-values are computed using binomial distributions and normal approximations. Recall, if a random variable $z \sim \text{Binomial}(N, p)$, then the probability p can be estimated by $\hat{p} = \frac{z}{N}$, and the variance of \hat{p} by $\hat{p}(1 - \hat{p})/N$.

Note that the test sets for *M-Noise1* to *M-Noise6* are very small as [10] did not intend to evaluate the statistical significance on those six datasets. (Private communications.)

Table 9: Summary of test *P*-values.

Dataset	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>
Covertype290k	3×10^{-10}	3×10^{-5}	9×10^{-8}	8×10^{-14}
Covertype145k	4×10^{-11}	4×10^{-7}	2×10^{-5}	7×10^{-9}
Poker525k	0	0	0	0
Poker275k	0	0	0	0
Poker150k	0	0	0	0
Poker100k	0	0	0	0
Poker25kT1	0	—	—	0
Poker25kT2	0	—	—	0
Mnist10k	5×10^{-8}	3×10^{-8}	1×10^{-7}	1×10^{-5}
M-Basic	2×10^{-4}	1×10^{-8}	1×10^{-5}	0.0164
M-Rotate	0	5×10^{-15}	6×10^{-11}	3×10^{-16}
M-Image	0	0	2×10^{-7}	7×10^{-7}
M-Rand	0	0	7×10^{-10}	8×10^{-4}
M-RotImg	0	0	2×10^{-6}	4×10^{-5}
M-Noise1	0.0029	0.0430	0.2961	0.0574
M-Noise2	0.0024	0.0072	0.1158	0.0583
M-Noise3	0.0190	0.0701	0.1073	0.0327
M-Noise4	0.0014	0.0090	0.4040	0.1935
M-Noise5	0.0188	0.0079	0.1413	0.2305
M-Noise6	0.0043	0.0058	0.1189	0.1002
Letter15k	0.0345	0.1718	0.1449	0.0268
Letter4k	2×10^{-6}	0.008	0.019	1×10^{-5}
Letter2k	2×10^{-5}	0.003	0.001	4×10^{-6}

These results demonstrate that *abc-logitboost* and *abc-mart* outperform *logitboost* and *mart*, respectively. In addition, except for *Poker25kT1* and *Poker25kT2*, *abc-logitboost* outperforms *abc-mart* and *logitboost* outperforms *mart*.

App. B provides more detailed *P*-values for *Mnsit10k* and *M-Image*, to demonstrate that the improvements hold for a wide range of parameters (J and ν).

4.6 Comparisons with SVM and Deep Learning

For *Poker* dataset, SVM could only achieve a test error rate of about 40% (Private communications with C.J. Lin). In comparison, all four algorithms, *mart*, *abc-mart*, (*robust*) *logitboost*, and *abc-logitboost*, could achieve much smaller error rates (i.e., < 10%) on *Poker25kT1* and *Poker25kT2*.

Fig. 2 provides the comparisons on the six (correlated) noise datasets: *M-Noise1* to *M-Noise6*, with SVM and deep learning based on the results in [10].

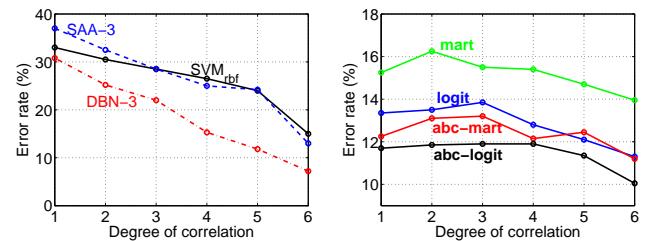


Figure 2: Six datasets: *M-Noise1* to *M-Noise6*. Left panel: Error rates of SVM and deep learning [10]. Right panel: Errors rates of four boosting algorithms. X-axis: degree of correlation from high to low; the values 1 to 6 correspond to the datasets *M-Noise1* to *M-Noise6*.

Table 10: Summary of test error rates of various algorithms on the modified *Mnist* dataset [10].

	M-Basic	M-Rotate	M-Image	M-Rand	M-RotImg
SVM-RBF	3.05%	11.11%	22.61%	14.58%	55.18%
SVM-POLY	3.69%	15.42%	24.01%	16.62%	56.41%
NNET	4.69%	18.11%	27.41%	20.04%	62.16%
DBN-3	3.11%	10.30%	16.31%	6.73%	47.39%
SAA-3	3.46%	10.30%	23.00%	11.28%	51.93%
DBN-1	3.94%	14.69%	16.15%	9.80%	52.21%
<i>mart</i>	4.12%	15.35%	11.64%	13.15%	49.82%
<i>abc-mart</i>	3.69%	13.27%	9.45%	10.60%	46.14%
<i>logitboost</i>	3.45%	13.63%	9.41%	10.04%	45.92%
<i>abc-logitboost</i>	3.20%	11.92%	8.54%	9.45%	44.69%

Table 10 compares the error rates on *M-Basic*, *M-Rotate*, *M-Image*, *M-Rand*, and *M-RotImg*, with the results in [10].

Fig. 2 and Table 10 illustrate that deep learning algorithms could produce excellent test results on certain datasets (e.g., *M-Rand* and *M-Noise6*). This suggests that there is still sufficient room for improvements in future research.

4.7 Test Errors versus Boosting Iterations

Again, we believe the plots for test errors versus boosting iterations could be more reliable than a single number, for comparing boosting algorithms.

Fig. 3 presents the test errors on *Mnist10k*, *M-Rand*, *M-Image*, *Letter15k*, *Letter4k*, and *Letter2k*. Recall we train the algorithms for up to $M = 10000$ iterations unless the training loss (3) is close to the machine accuracy.

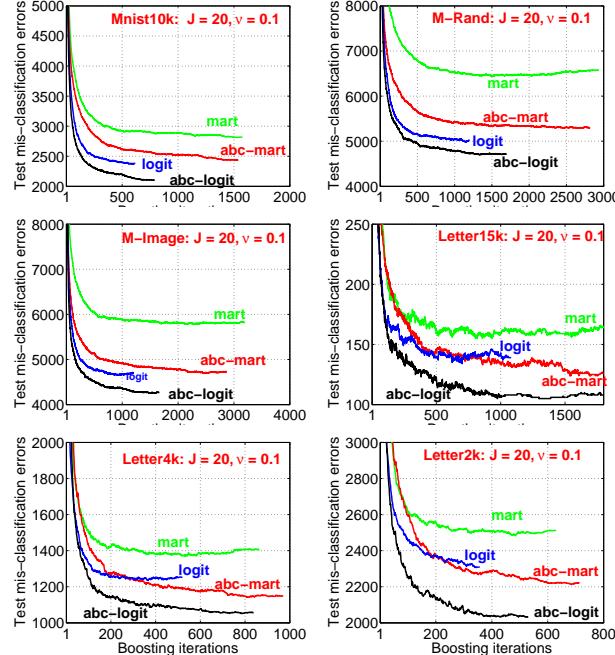


Figure 3: Test mis-classification errors on *Mnist10k*, *M-Rand*, *M-Image*, *Letter15k*, *Letter4k*, and *Letter2k*.

Fig. 4 provides the test mis-classification errors on various datasets from *Covertype* and *Poker*. For these large datasets, we only train $M = 5000$ iterations. (The machine accuracy is not reached.)

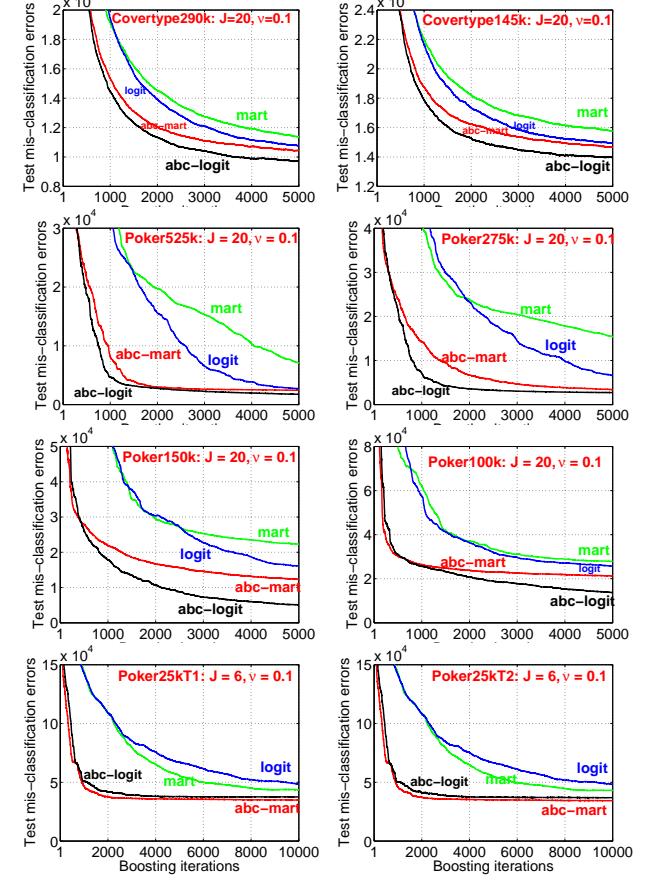


Figure 4: Test mis-classification errors on various datasets of *Covertype* and *Poker*.

4.8 Relative Improvements versus Boosting Iterations

For certain applications, it may not be always affordable to use very large models (i.e., many boosting iterations) in the test phrase. Fig. 5 reports the relative improvements (*abc-logitboost* over (*robust*) *logitboost* and *abc-mart* over *mart*) of the test errors versus boosting iterations.

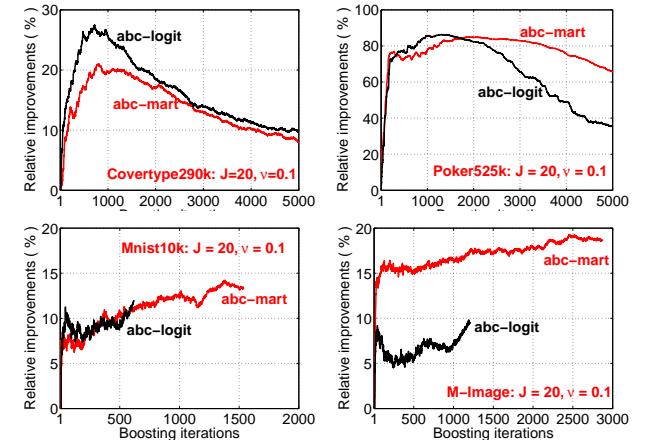


Figure 5: Relative improvements (%) of test errors on *M-Image*, *Letter15k*, *Letter4k*, and *Letter2k*.

5 Conclusion

Classification is a fundamental task in statistics and machine learning. This paper presents *robust logitboost* and *abc-logitboost*, with extensive experiments.

Robust logitboost provides the explicit formulation of the tree-split criterion for implementing the influential *logitboost* algorithm. *Abc-logitboost* is developed for multi-class classification, by combining (*robust*) *logitboost* with *abc-boost*, a new boosting paradigm proposed by [12]. Our extensive experiments demonstrate its superb performance.

We also compare our boosting algorithms with a variety of learning methods including SVM and *deep learning*, using the results in prior publications, e.g., [10]. For certain datasets, *deep learning* obtained adorable performance that our current boosting algorithms could not achieve, suggesting there is still room for improvement in future research.

Acknowledgement

The author is indebted to Professor Friedman and Professor Hastie for their encouragements and suggestions on this line of work. The author thanks C.J. Lin for the SVM experiments on the *Poker* dataset. The author also thanks L. Bottou and D. Erhan for the explanations on the datasets used in their papers.

The author is partially supported by NSF (DMS-0808864), ONR (N000140910911, Young Investigator Award), and the “Beyond Search - Semantic Computing and Internet Economics” Microsoft 2007 Award.

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A Experiments on Binary Classification

Table 11 lists four datasets for binary classification, to compare *robust logitboost* with *mart*. Fig. 6 reports the results.

Table 11: Datasets for binary classification experiments

dataset	K	# training	# test	# features
Mnist2Class	2	60000	10000	784
IJCNN1	2	49990	91701	22
Forest521k	2	521012	50000	54
Forest100k	2	100000	50000	54

Forest521k and **Forest100k** were the two largest datasets in a fairly recent SVM paper [2]. **Mnist2Class** converted the original 10-class MNIST dataset into a binary problem by combining digits from 0 to 4 as one class and 5 to 9 as another class. **IJCNN1** was used in a competition. The winner used SVM (see page 8 at http://www.geocities.com/ijcnn/nnc_ijcnn01.pdf).

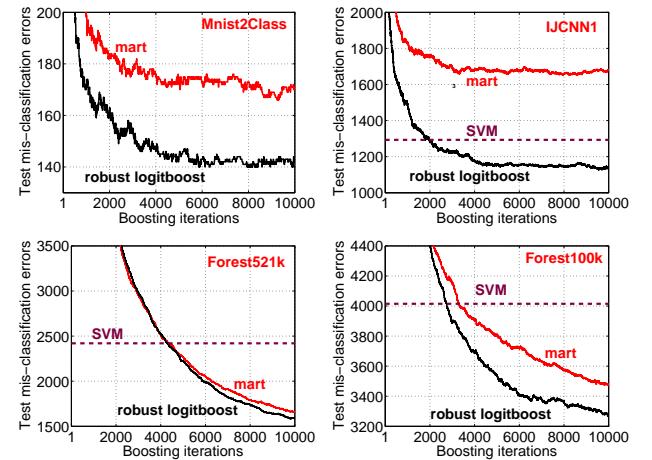


Figure 6: Test mis-classification errors for binary classification. In all experiments, we always use the tree size $J = 20$ and the shrinkage $\nu = 0.1$.

B P-values for the Experiments on *Mnist10k* and *M-Image*

See Sec. 4.5 for the definitions of P1, P2, P3, and P4. We compute the P -values for all combinations of parameters, to show that the improvements are significant not just for one particular set of parameters.

Table 12: *Mnist10k*: P -values.

P1			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	7×10^{-5}	3×10^{-5}	7×10^{-10}
$J = 6$	8×10^{-9}	1×10^{-10}	9×10^{-11}
$J = 8$	9×10^{-12}	4×10^{-12}	5×10^{-13}
$J = 10$	4×10^{-11}	2×10^{-10}	4×10^{-11}
$J = 12$	1×10^{-9}	7×10^{-11}	1×10^{-10}
$J = 14$	6×10^{-10}	1×10^{-9}	6×10^{-9}
$J = 16$	2×10^{-9}	3×10^{-10}	6×10^{-9}
$J = 18$	3×10^{-8}	2×10^{-9}	6×10^{-10}
$J = 20$	2×10^{-8}	3×10^{-8}	2×10^{-8}
$J = 24$	2×10^{-8}	1×10^{-8}	6×10^{-8}
$J = 30$	1×10^{-7}	5×10^{-8}	2×10^{-7}
$J = 40$	3×10^{-7}	1×10^{-7}	2×10^{-8}
$J = 50$	6×10^{-6}	1×10^{-7}	3×10^{-7}

P2			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	2×10^{-8}	2×10^{-6}	6×10^{-6}
$J = 6$	1×10^{-10}	4×10^{-8}	9×10^{-9}
$J = 8$	4×10^{-10}	2×10^{-9}	1×10^{-10}
$J = 10$	7×10^{-11}	4×10^{-10}	3×10^{-11}
$J = 12$	1×10^{-10}	2×10^{-10}	2×10^{-11}
$J = 14$	2×10^{-11}	8×10^{-12}	2×10^{-10}
$J = 16$	1×10^{-11}	8×10^{-11}	7×10^{-12}
$J = 18$	5×10^{-11}	9×10^{-12}	6×10^{-12}
$J = 20$	2×10^{-10}	2×10^{-9}	1×10^{-9}
$J = 24$	1×10^{-8}	3×10^{-9}	3×10^{-8}
$J = 30$	2×10^{-7}	2×10^{-8}	5×10^{-9}
$J = 40$	3×10^{-5}	1×10^{-5}	4×10^{-6}
$J = 50$	0.0026	0.0023	3×10^{-4}

P3			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	3×10^{-9}	5×10^{-9}	4×10^{-6}
$J = 6$	4×10^{-13}	2×10^{-8}	2×10^{-10}
$J = 8$	2×10^{-9}	3×10^{-10}	3×10^{-10}
$J = 10$	1×10^{-10}	8×10^{-10}	6×10^{-11}
$J = 12$	2×10^{-10}	2×10^{-8}	1×10^{-9}
$J = 14$	5×10^{-10}	6×10^{-9}	4×10^{-10}
$J = 16$	2×10^{-8}	2×10^{-7}	1×10^{-8}
$J = 18$	4×10^{-9}	8×10^{-9}	6×10^{-8}
$J = 20$	1×10^{-6}	2×10^{-7}	6×10^{-8}
$J = 24$	2×10^{-5}	9×10^{-6}	3×10^{-6}
$J = 30$	5×10^{-4}	0.0011	1×10^{-4}
$J = 40$	0.0056	0.0103	0.0024
$J = 50$	0.0145	0.0707	0.0218

P4			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	1×10^{-5}	2×10^{-7}	4×10^{-10}
$J = 6$	5×10^{-11}	7×10^{-11}	1×10^{-12}
$J = 8$	4×10^{-11}	5×10^{-13}	2×10^{-12}
$J = 10$	6×10^{-11}	5×10^{-10}	8×10^{-11}
$J = 12$	2×10^{-9}	6×10^{-9}	6×10^{-8}
$J = 14$	1×10^{-8}	4×10^{-7}	1×10^{-8}
$J = 16$	1×10^{-8}	4×10^{-7}	9×10^{-9}
$J = 18$	1×10^{-6}	5×10^{-7}	3×10^{-6}
$J = 20$	4×10^{-5}	8×10^{-7}	2×10^{-6}
$J = 24$	3×10^{-5}	3×10^{-5}	7×10^{-6}
$J = 30$	3×10^{-4}	0.0016	0.0012
$J = 40$	2×10^{-4}	5×10^{-4}	6×10^{-5}
$J = 50$	9×10^{-5}	7×10^{-5}	2×10^{-4}

When the author presented this work at various seminars, several researchers were curious about the good performance of our boosting algorithms on the *M-Image* dataset. Thus, we would like to provide more details of the experiments on this dataset.

Table 13: *M-Image*: P -values.

P1			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	7×10^{-10}	1×10^{-10}	3×10^{-11}
$J = 6$	5×10^{-12}	6×10^{-13}	1×10^{-13}
$J = 8$	1×10^{-13}	0	0
$J = 10$	0	0	0
$J = 12$	0	0	0
$J = 14$	0	0	0
$J = 16$	0	0	0
$J = 18$	0	0	0
$J = 20$	0	0	0
$J = 24$	0	0	0
$J = 30$	0	0	0
$J = 40$	0	0	0
$J = 50$	0	0	0

P2			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	1×10^{-10}	1×10^{-9}	1×10^{-9}
$J = 6$	5×10^{-12}	3×10^{-11}	1×10^{-11}
$J = 8$	0	0	0
$J = 10$	0	0	0
$J = 12$	0	0	0
$J = 14$	0	0	0
$J = 16$	0	0	0
$J = 18$	0	0	0
$J = 20$	0	0	0
$J = 24$	0	0	0
$J = 30$	0	0	0
$J = 40$	0	0	0
$J = 50$	0	0	0

P3			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	0.001	8×10^{-4}	0.0003
$J = 6$	6×10^{-5}	7×10^{-7}	6×10^{-6}
$J = 8$	8×10^{-9}	2×10^{-7}	1×10^{-6}
$J = 10$	2×10^{-8}	3×10^{-8}	4×10^{-9}
$J = 12$	2×10^{-8}	1×10^{-7}	6×10^{-8}
$J = 14$	6×10^{-8}	2×10^{-8}	3×10^{-6}
$J = 16$	1×10^{-8}	8×10^{-8}	2×10^{-8}
$J = 18$	1×10^{-8}	2×10^{-8}	5×10^{-8}
$J = 20$	7×10^{-8}	9×10^{-8}	8×10^{-8}
$J = 24$	1×10^{-7}	6×10^{-5}	2×10^{-6}
$J = 30$	8×10^{-4}	3×10^{-4}	4×10^{-4}
$J = 40$	0.0254	0.0133	0.0475
$J = 50$	0.0356	0.0818	0.0354

P4			
	$\nu = 0.04$	$\nu = 0.06$	$\nu = 0.08$
$J = 4$	0.0025	0.0002	3×10^{-5}
$J = 6$	5×10^{-5}	4×10^{-8}	3×10^{-7}
$J = 8$	8×10^{-9}	2×10^{-7}	1×10^{-6}
$J = 10$	2×10^{-8}	1×10^{-8}	3×10^{-8}
$J = 12$	9×10^{-10}	3×10^{-9}	2×10^{-9}
$J = 14$	1×10^{-8}	5×10^{-10}	5×10^{-9}
$J = 16$	5×10^{-9}	1×10^{-8}	8×10^{-9}
$J = 18$	6×10^{-7}	1×10^{-7}	1×10^{-8}
$J = 20$	5×10^{-7}	5×10^{-8}	2×10^{-7}
$J = 24$	2×10^{-6}	4×10^{-4}	2×10^{-6}
$J = 30$	7×10^{-4}	8×10^{-5}	3×10^{-4}
$J = 40$	0.0017	9×10^{-4}	4×10^{-4}
$J = 50$	0.0032	0.0033	7×10^{-4}