Convergent and Correct Message Passing Schemes for Optimization Problems over Graphical Models

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Abstract

The max-product algorithm, which attempts to compute the most probable assignment (MAP) of a given probability distribution, has recently found applications in quadratic minimization and combinatorial optimization. Unfortunately, the max-product algorithm is not guaranteed to converge and, even if it does, is not guaranteed to produce the MAP assignment. In this work, we provide a simple derivation of a new family of message passing algorithms by "splitting" the factors of our graphical model. We prove that, for any objective function that attains its maximum value over its domain, this new family of message passing algorithms always contains a message passing scheme that guarantees correctness upon convergence to a unique estimate. Finally, we adopt an asynchronous message passing schedule and prove that, under mild assumptions, such a schedule guarantees the convergence of our algorithm.

1 Introduction

In general, computing the MAP assignment is an NP-hard problem, but for graphical models possessing a single cycle the algorithm is guaranteed to converge to the MAP assignment under a few mild assumptions [1]. Over arbitrary graphical models, the max-product algorithm may fail to converge or, worse, may converge to an assignment that is not the MAP. Despite these difficulties, max-product and its variants have found empirical success in a variety of application areas including statistical physics, combinatorial optimization, computer vision, clustering, error-correcting codes, and the minimization of convex functions; however, rigorously characterizing their behavior outside of a few well-structured instances has proved challenging.

We propose a new message passing scheme for the solving the MAP problem based on a simple "splitting" heuristic. Unlike other convergent message passing schemes (for example, [2]), the derivation of this algorithm is surprisingly simple, and the update rules closely mirror the standard max-product message updates. Because of its simplicity, we are able to present the algorithm in its most general form: our algorithm is not restricted to binary state spaces or pairwise factor graphs. More importantly, almost all of the intuition for the standard max-product algorithm can be extended with very little effort to our framework.

Our algorithm requires choosing a set of constants, but any choice of non-zero constants will suffice to produce a valid message passing algorithm. In this way, our message passing scheme is more appropriately thought of as a family of message passing algorithms. We show that, under a mild assumption, there is always a simple choice of constants that will guarantee convergence. Further, if we are able to extract a unique estimate from the converged beliefs then this estimate is guaranteed to be the MAP assignment.

The details of this work are available online [3].

References

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