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# Theoretical Foundations, Numerical Techniques and Application of Transport Coefficients in Modeling of Streamers and Sprites

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Centrum Wiskunde & Informatica, Amsterdam, The Netherlands

Workshop on TLE's and TGF's, topical meeting of E-CANES, ASIM and TARANIS, Amsterdam, October 25-27, 2010

# Outline of Talk



## Introduction

- What is a swarm of charged particles?
- What can swarms bring to modeling of streamers and sprites?

## How do we solve the Boltzmann equation?

- Two term approximation vs. Multi term theory;
- Duality of transport coefficients.

## Transport Coefficients in Gases

- Three-body attachment in $O_2$ and pressure dependence of transport coefficients;
- Explicit and implicit effects of non-conservative collisions on electron transport coefficients;
- Electron kinetic processes related to discharges in the atmosphere of Venus;
- Electron kinetic processes related to discharges in the atmospheres of gas giants;
- Modeling electron transport in magnetized planetary atmospheric discharges.

## Conclusion and the Future

# What is a swarm of charged particles?

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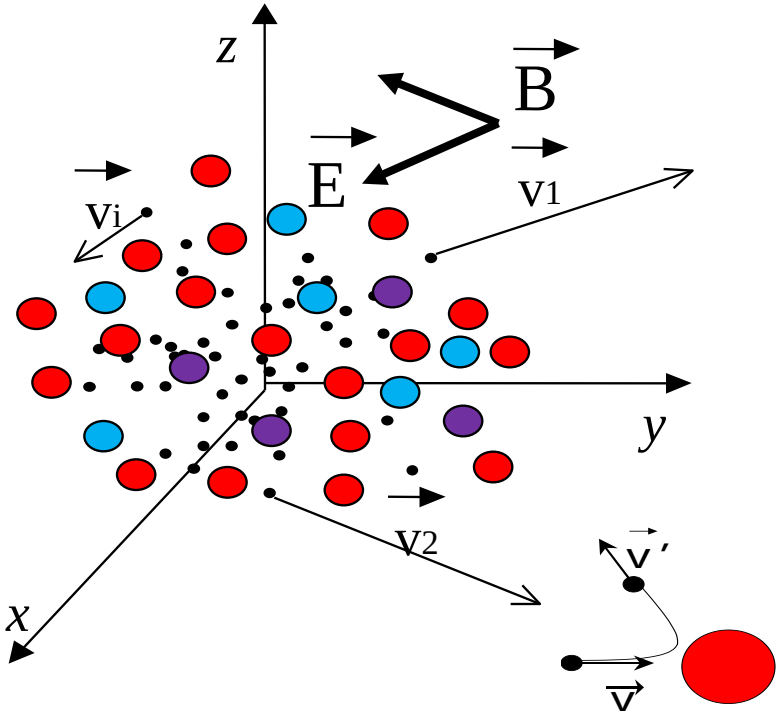
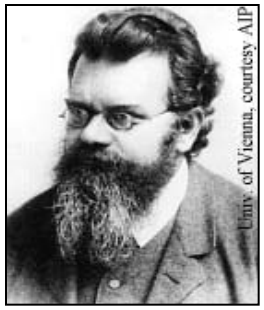
**Swarm conditions ≡ Free diffusion plasma limit**




Ludwig Boltzmann (1844-1906)

## Boltzmann equation:

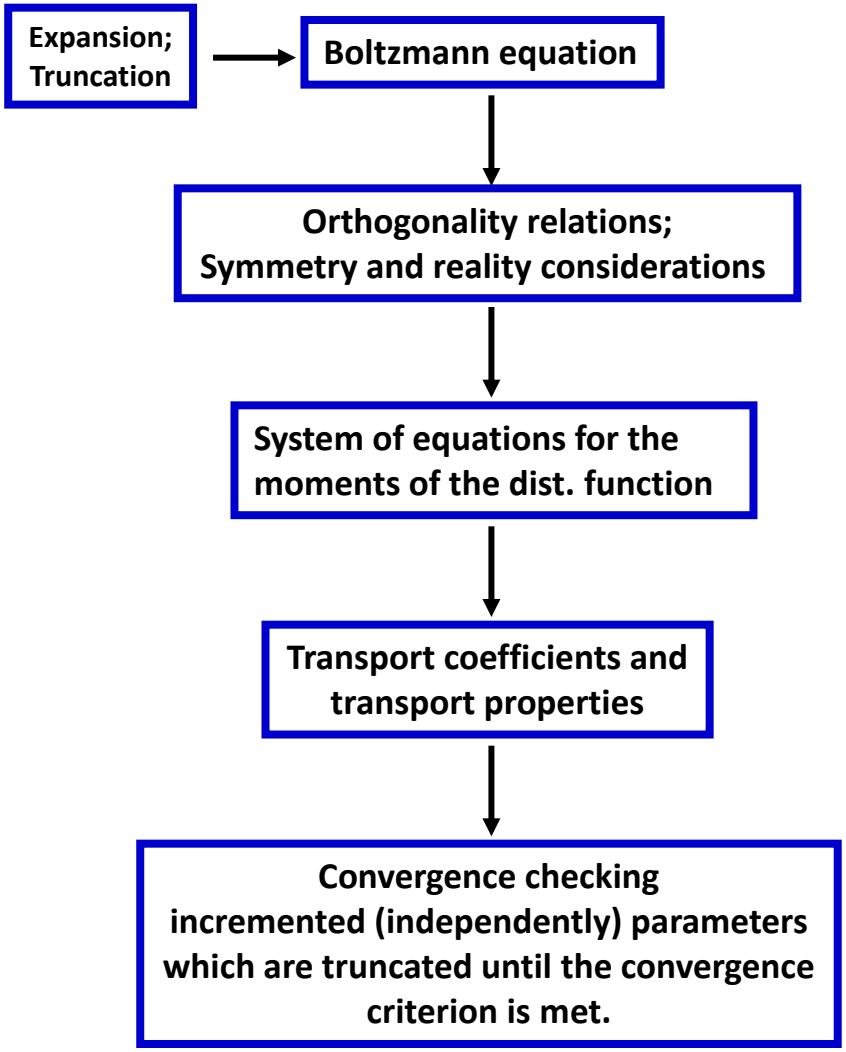
$$\frac{\partial f}{\partial t} + \mathbf{c}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{q}{m}\left(\mathbf{E} + \mathbf{c}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{c}} = -J\left(f, F_0\right)$$

$f(r,c,t)$ - phase space distribution function;
r - space co-ordinate;
c - velocity co-ordinate;
$E$, $B$ - electric and magnetic field strengths;
$J(f,F_0)$ - collision operator.

- Low density of charged particle in gases:
  - neglect charged particle – charged particle interaction;
  - neglect space charge effects;
- $E$ and $B$ fields are spatially homogeneous and externally prescribed;
- Small spatial gradients in number density;
- Minimal boundary effects.

# How do we solve the Boltzmann equation?






- Resolving the angular dependence in velocity space:

$$f(\mathbf{r},\mathbf{c},t)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l} f_m^{(l)}(\mathbf{r},c,t)Y_m^{[l]}(\hat{\mathbf{c}})$$

- Projecting out the space dependence:
  - Hydrodynamic regime:

$$f_m^{(l)}(\mathbf{r},c,t)=\sum_{s=0}^{\infty}\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda} f(lm\,|\,s\lambda\mu;c,t)G_{\mu}^{(s\lambda)}n(\mathbf{r},t)$$

  - Non-hydrodynamic regime:
    - finite difference
    - pseudo-spectral

- Resolving the speed dependence:

$$f(lm\,|\,s\lambda\mu;c,t)=\omega(\alpha,c)\sum_{\nu=0}^{\infty}F(\nu lm\,|\,s\lambda\mu;\alpha,t)R_{\nu l}(\alpha c)$$

Dujko *et al.* Phys. Rev. E **81** (2010) 046403

# Connection: Theory & Experiment



- **Bulk Transport Coefficients**
  - Measured in swarm experiments
  - Determined according to the diffusion equation

$$\frac{\partial n}{\partial t} + \mathbf{W}(t).\nabla n - \mathbf{D}(t) : \nabla\nabla n = -R_a(t)$$

where

$\mathbf{W}$ = bulk drift velocity

$\mathbf{D}$ = bulk diffusion tensor

$R_a$ = net loss rate

- **Flux Transport Coefficients**
  - Unmeasurable!
  - Required in fluid models
  - Determined via flux-gradient relation

$$\Gamma(r,t) = n(r,t)\mathbf{W}^{(*)}(t) - \mathbf{D}^{(*)}(t).\nabla n$$

where

$\mathbf{W}^{(*)}$ = flux drift velocity

$\mathbf{D}^{(*)}$ = flux diffusion tensor

- **Bulk and flux vary only in the presence of non-conservative collisions;**
- **Differences are often significant, from a few % to a few orders of magnitude;**
- **Relationship exists to convert between the two sets;**
- **Fluid modelers should be aware of the origin of the transport data they are using.**

# The influence of 3-body attachment on electron transport

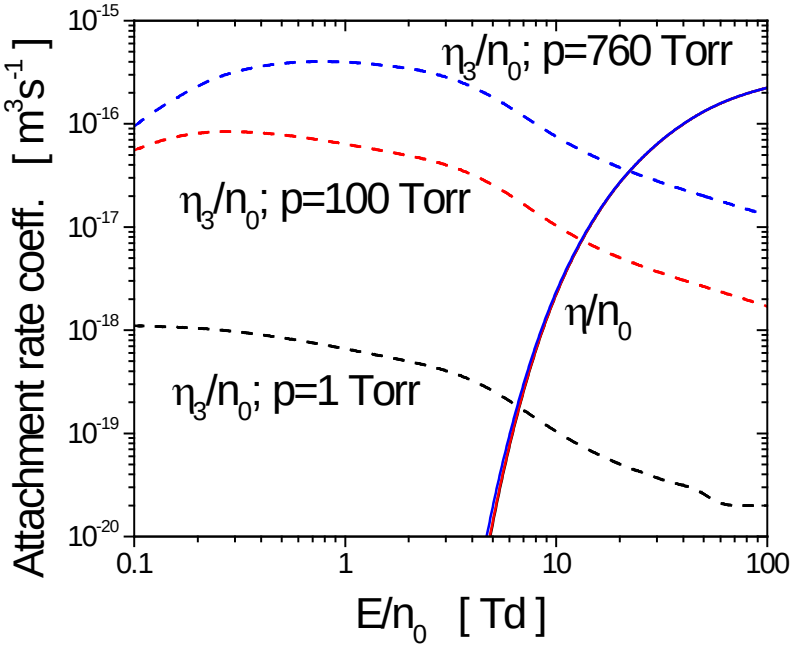
$$O_2 + e \rightarrow O_2^{-*} \quad (1)$$

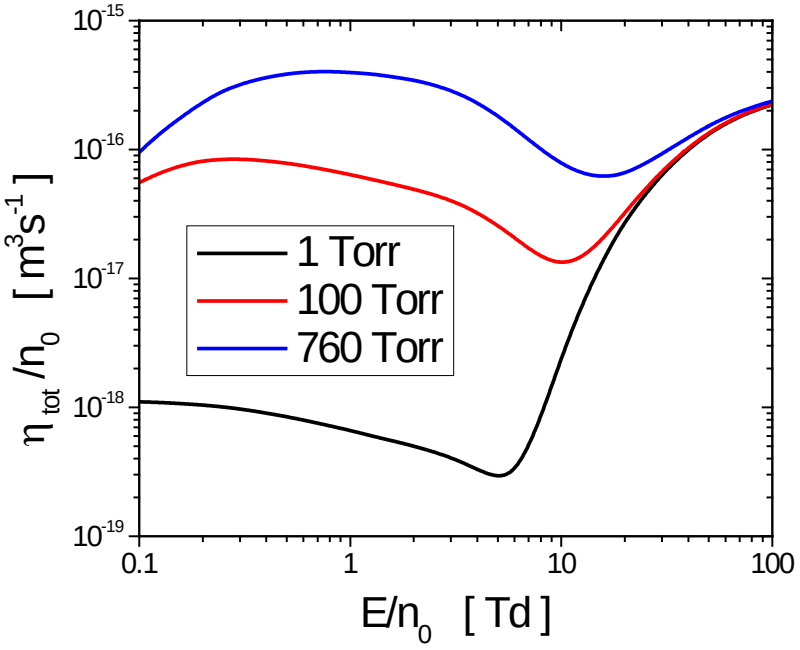
(1) →

$$O_2^{-*} \rightarrow O_2(v) + e \quad \text{(auto-detachment)} \quad (2)$$

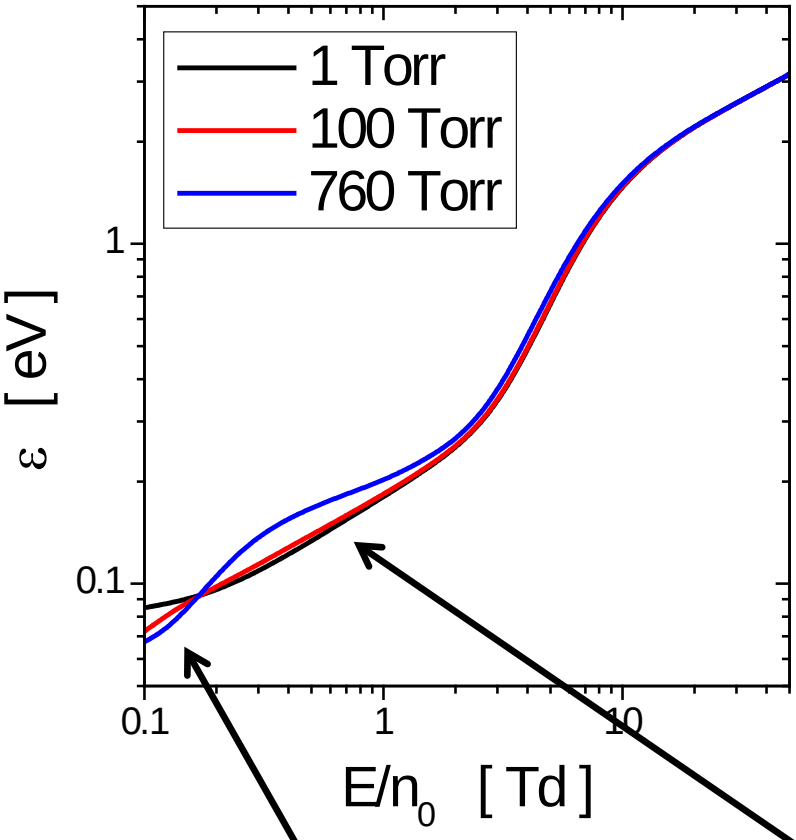
(2) →

$$O_2^{-*} + O_2 \rightarrow O_2^{-}(\text{stable}) + O_2 \quad (3)$$

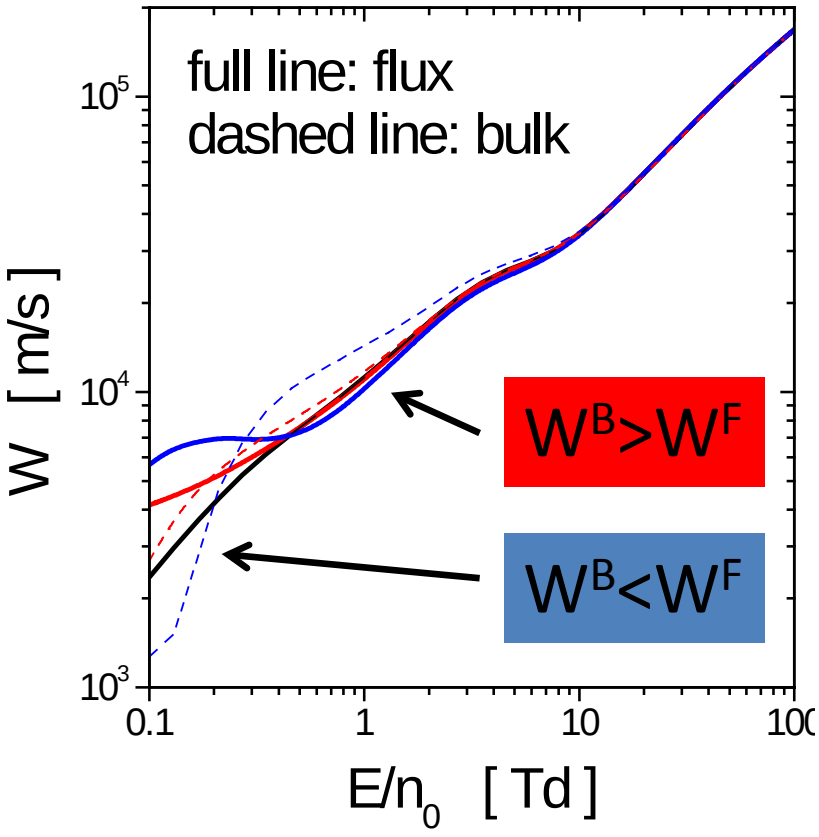


## Mean energy



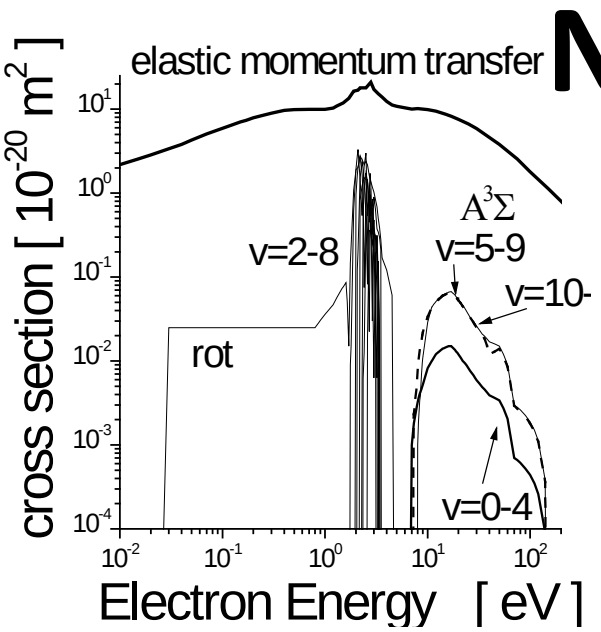

## Drift velocity

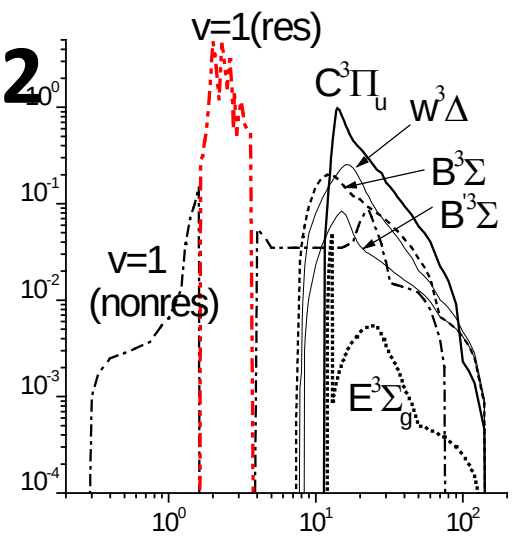


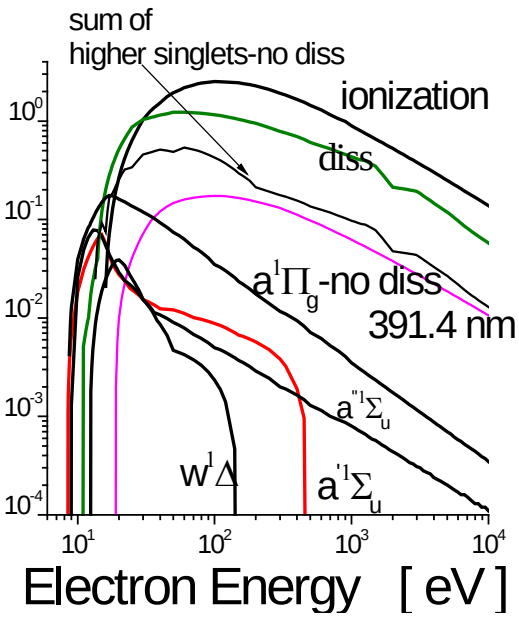

attachment cooling

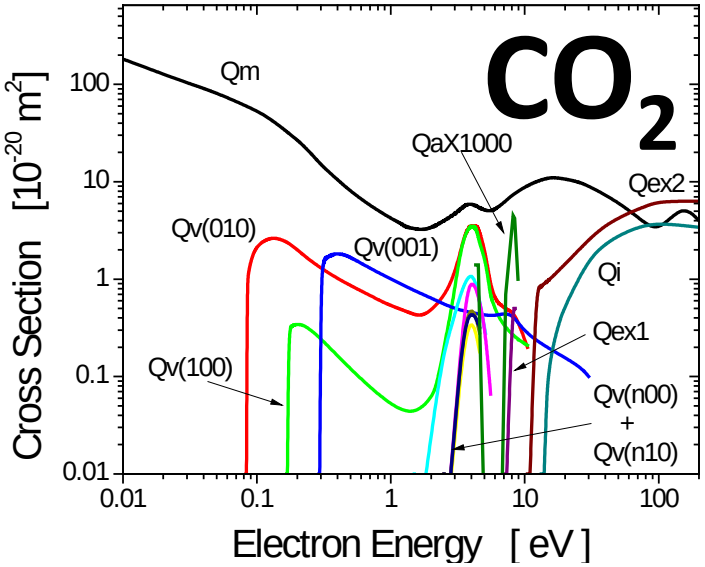
attachment heating

# Electron kinetic processes related to discharges in the atmosphere of Venus




**Venus: 96.5% $CO_2$ : 3.5% $N_2$**

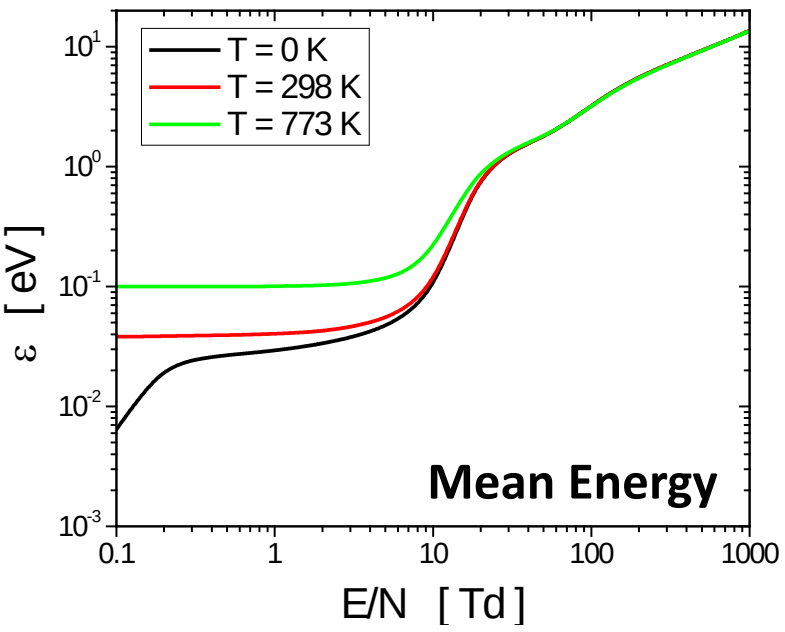
**$N_2$**: Gaseous Electronics Laboratory, Institute of Physics, University of Belgrade.

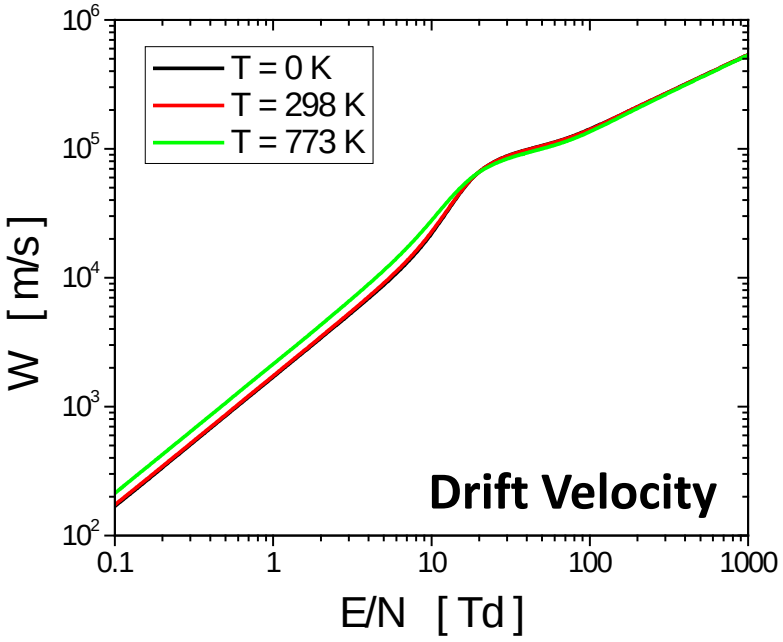
**$CO_2$**: Keio University, Japan

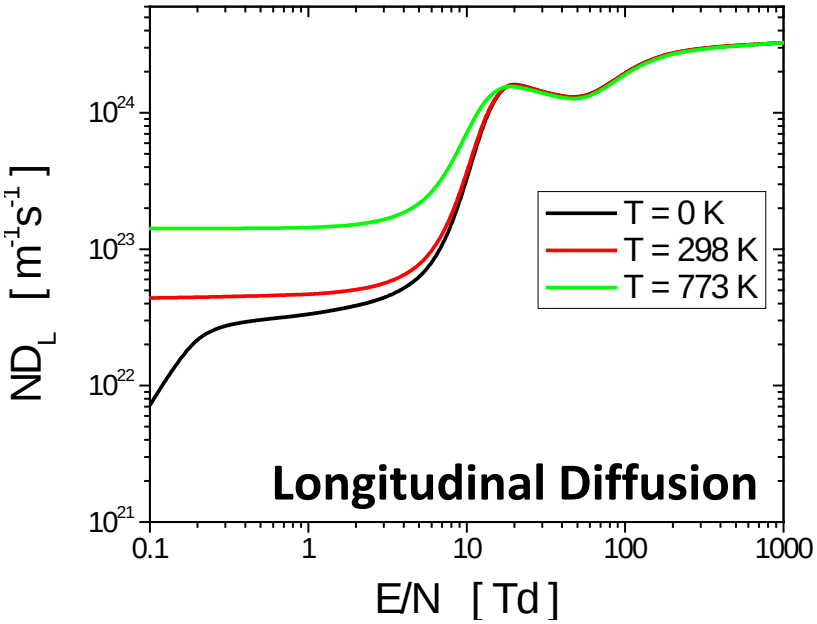
Y. Nakamura, Aust. J. Phys., 1995, **48**, 357-363

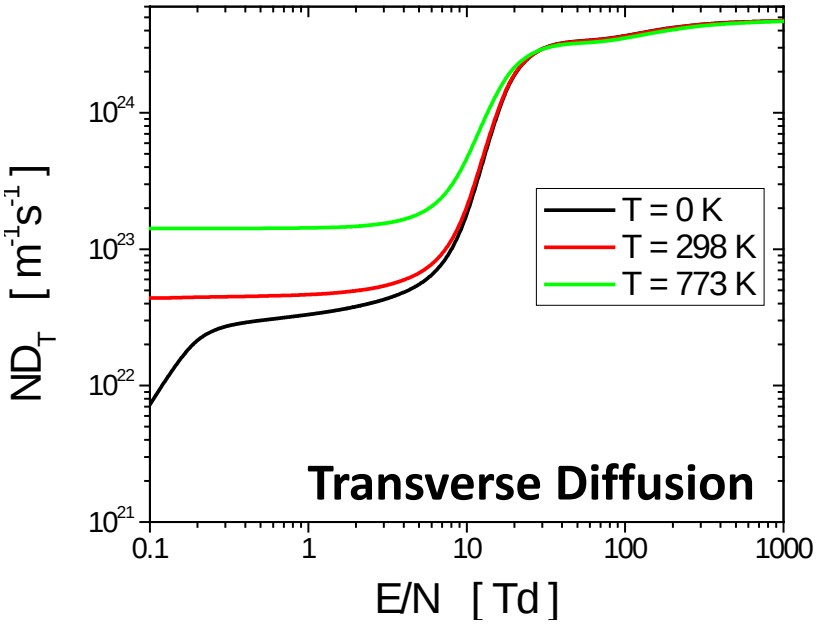
# Electron kinetic processes related to discharges in the atmosphere of Venus



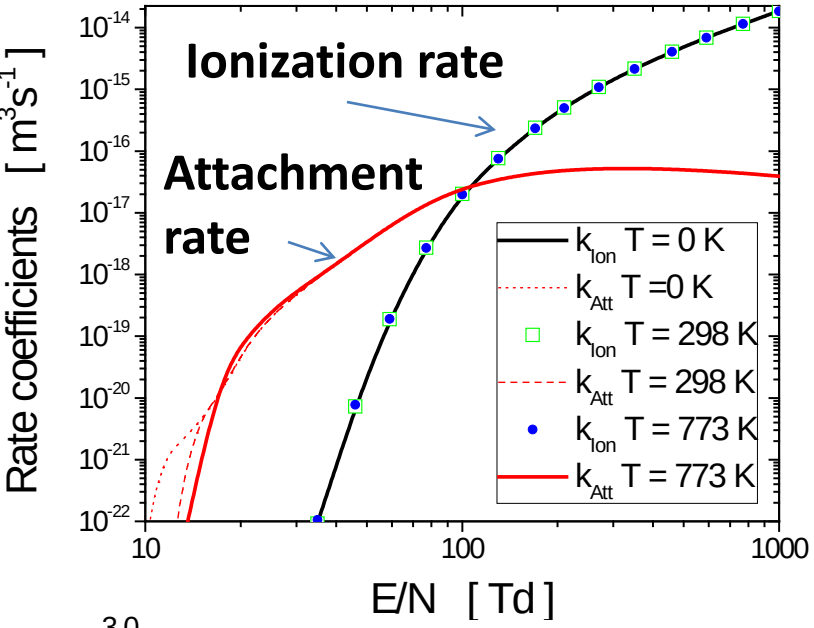


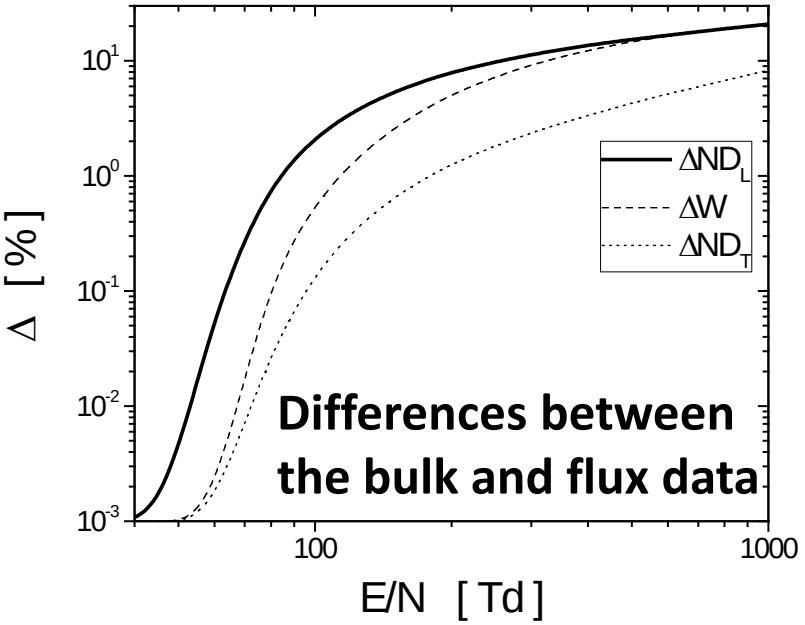



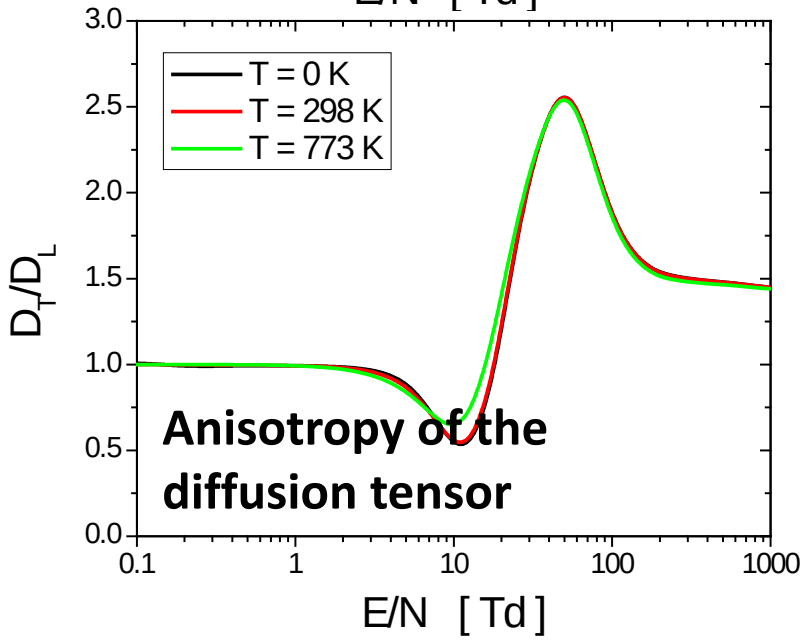



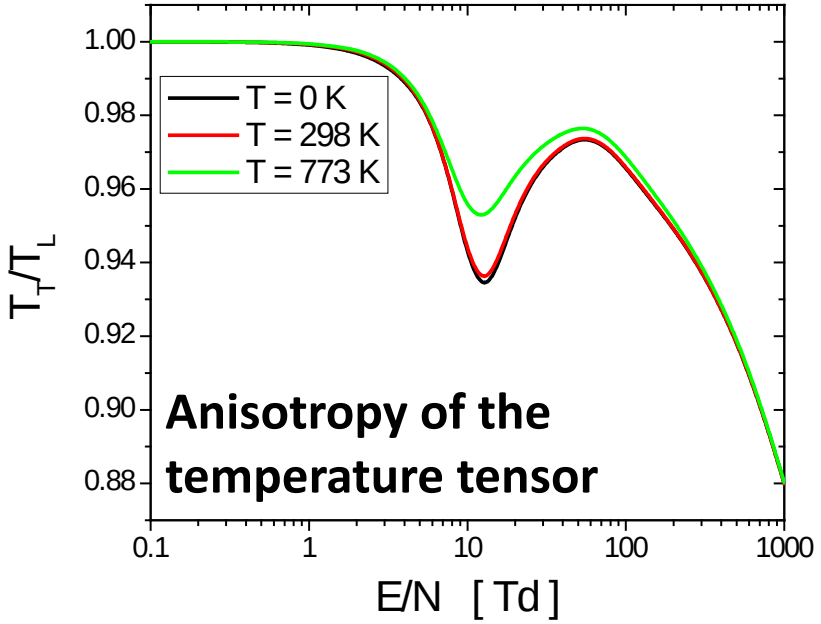



# Electron kinetic processes related to discharges in the atmosphere of Venus



Electron kinetic processes related to discharges in atmospheres of gas giants

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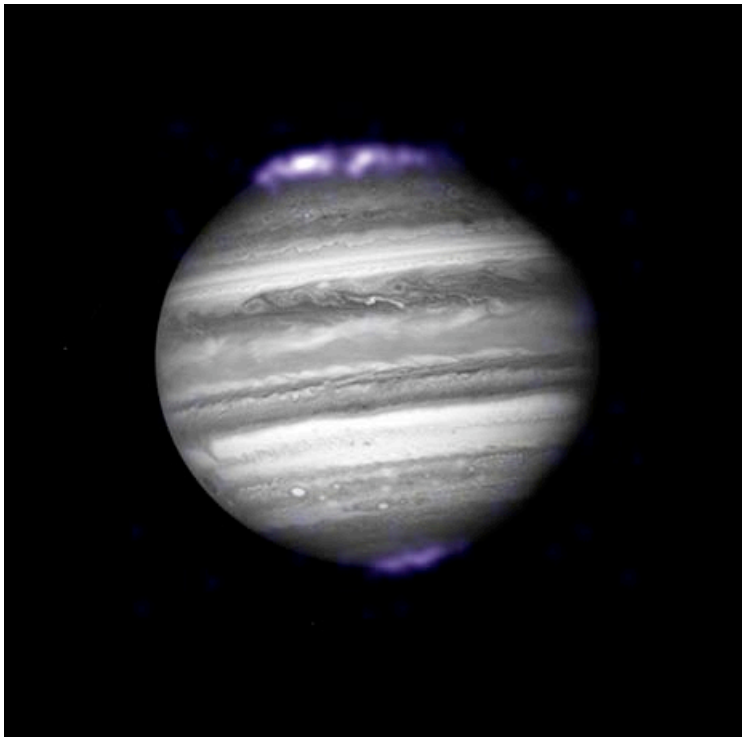
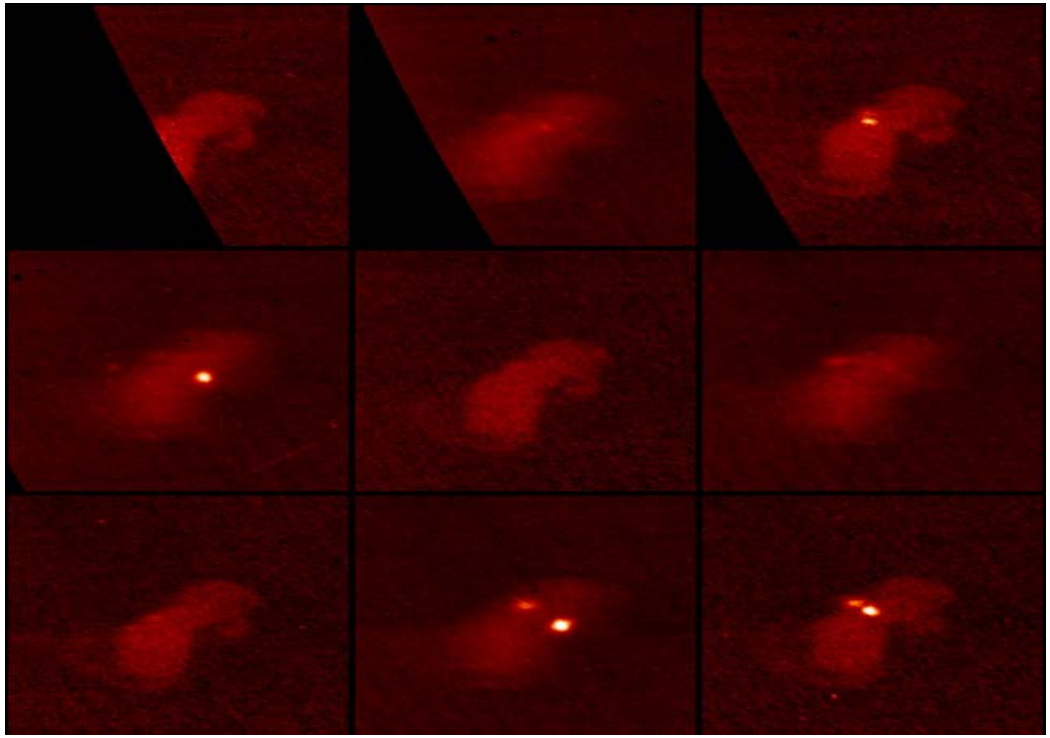
# Jupiter Auroras



The first polar lightning on a non-terrestrial planet was detected by the New Horizons spacecraft in 2007!

# Lightning on Saturn



NASA's Cassini spacecraft captured the first lightning flashes on Saturn when it captured these images on Aug. 17, 2009!

# Electron kinetic processes related to discharges in atmospheres of gas giants

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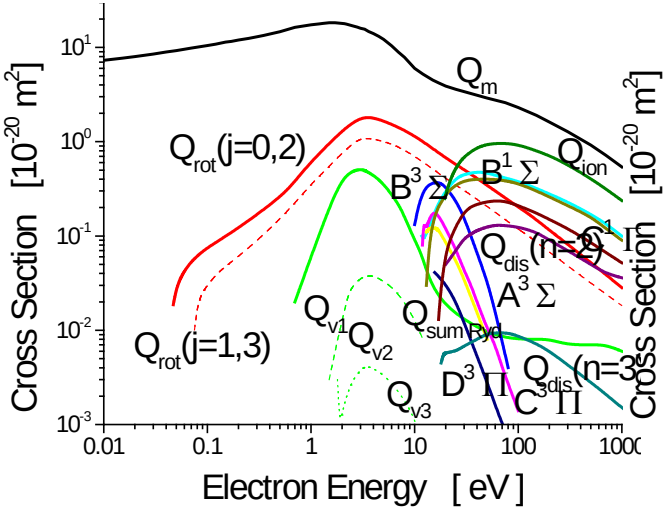
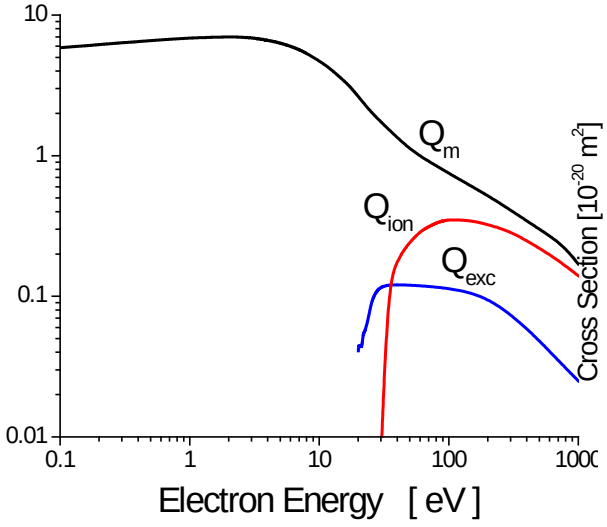
| Planet | Gas Mixture |
|---|---|
| Jupiter | 89% $H_2$, 10.9% He, 0.1% $CH_4$ |
| Saturn | 92.1% $H_2$, 7.4% He, 0.5% $CH_4$ |
| Uranus | 82.5% $H_2$, 15.2% He, 5% $CH_4$ |
| Neptune | 80% $H_2$, 18.5% He, 1.5% $CH_4$ |
| Titan | 95% $N_2$, 5% $CH_4$ |

*Relative concentrations of the constituent gases that make up the atmosphere's of the gas giants and Titan.*

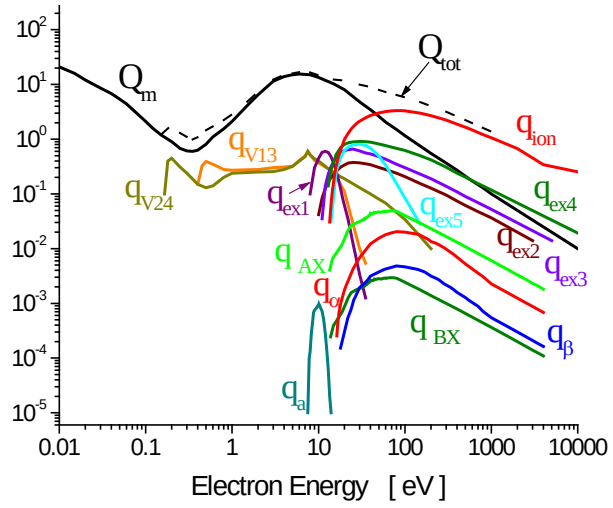
## $H_2$




## He

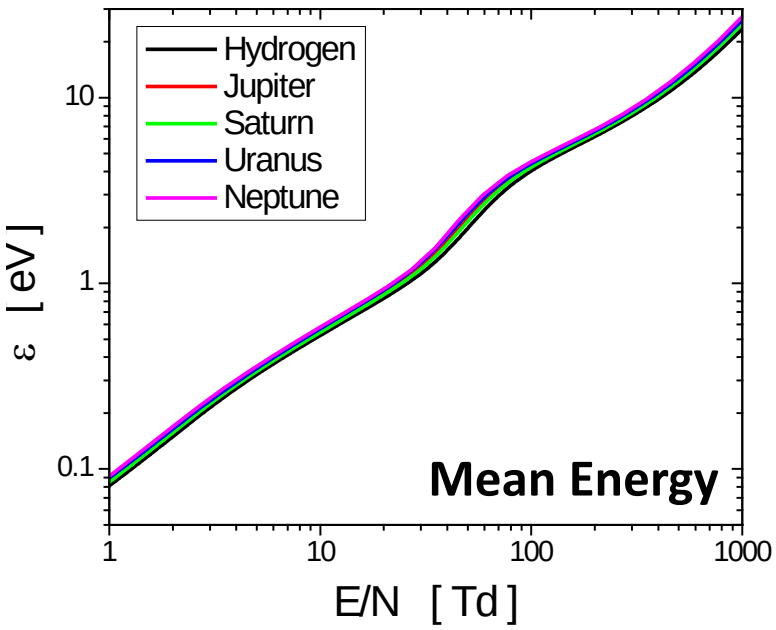


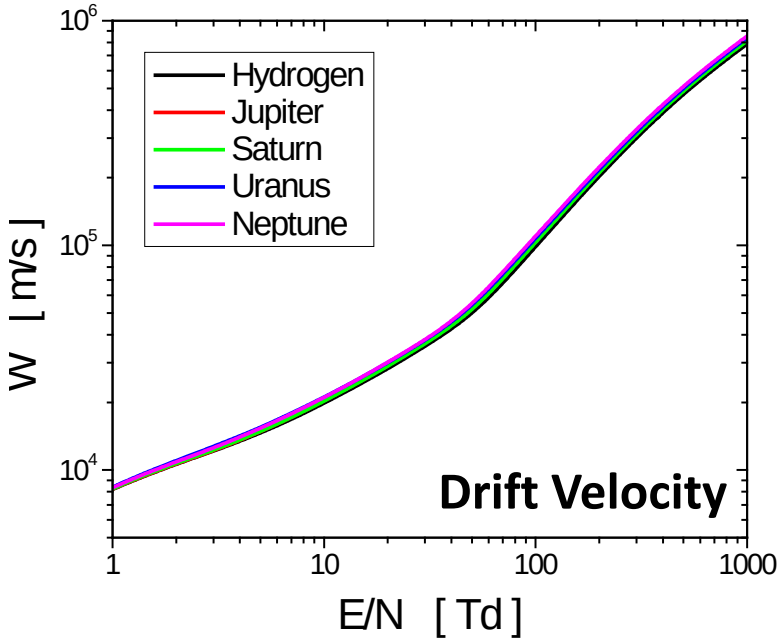

## $CH_4$

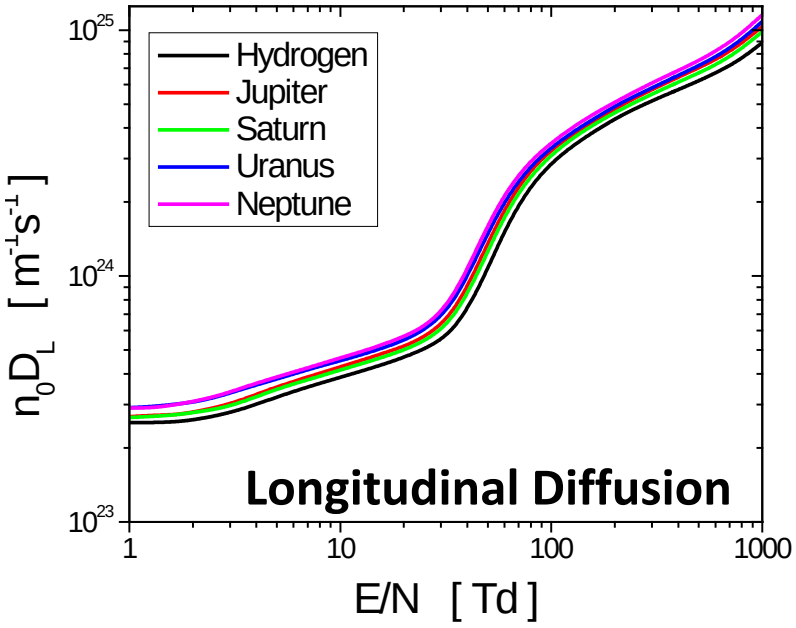


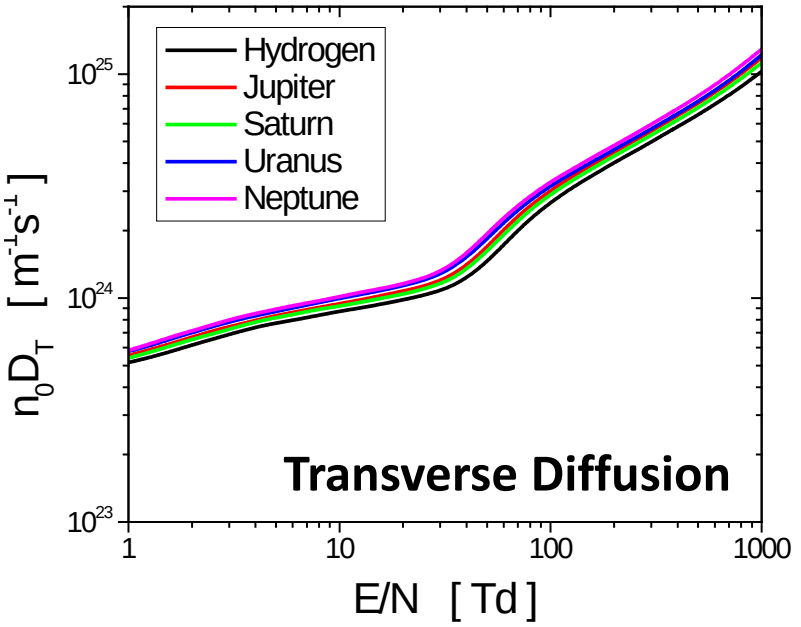
# Electron kinetic processes related to discharges in atmospheres of gas giants





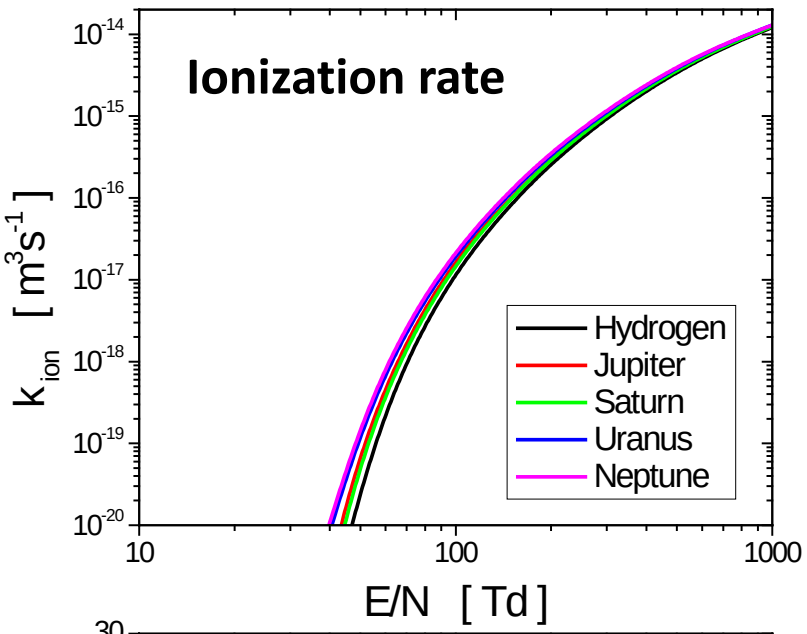



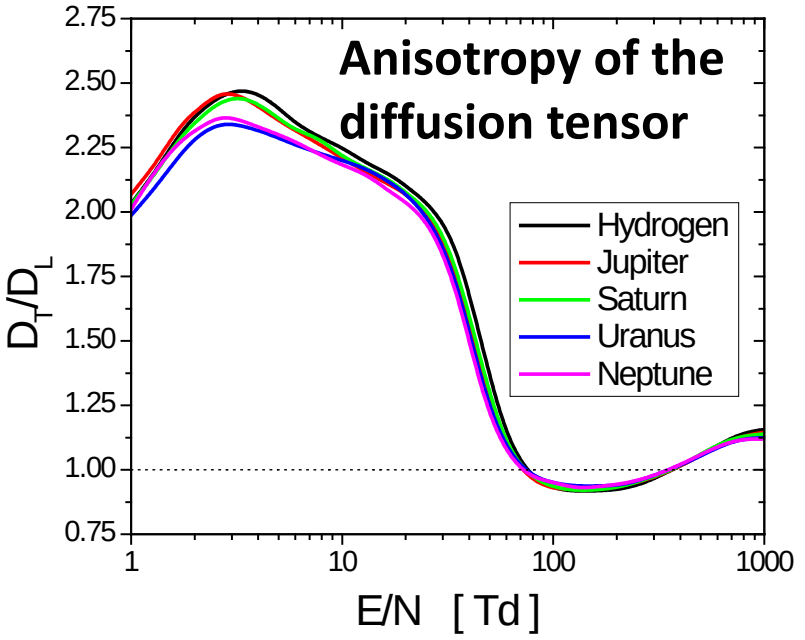



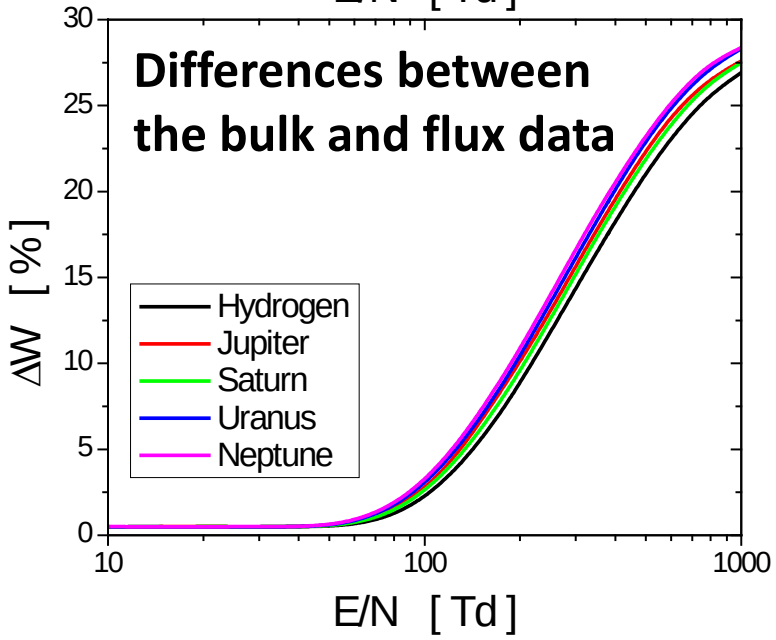



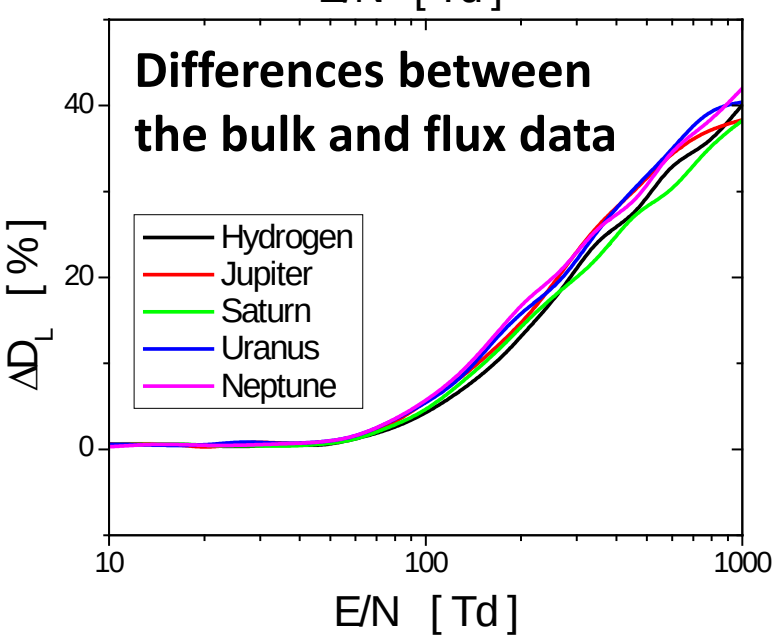
# Electron kinetic processes related to discharges in atmospheres of gas giants





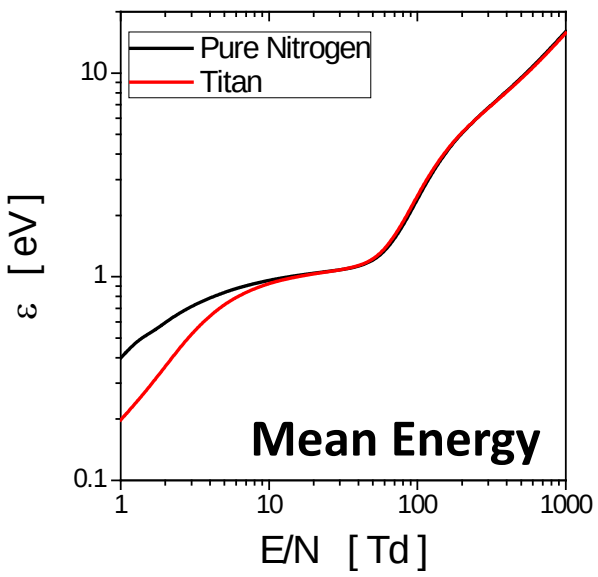



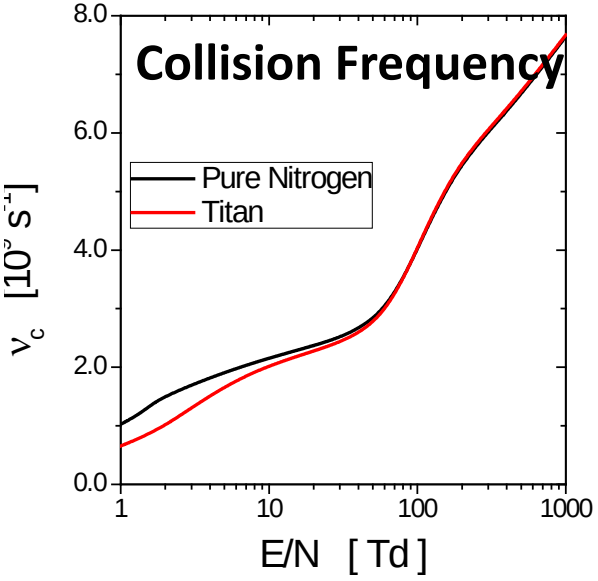



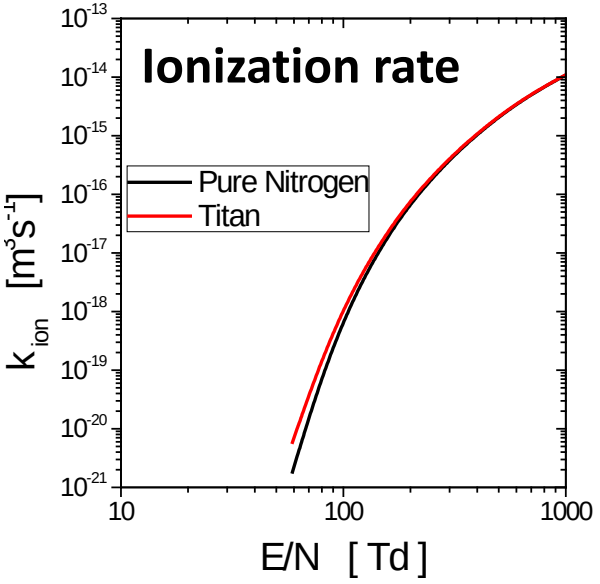



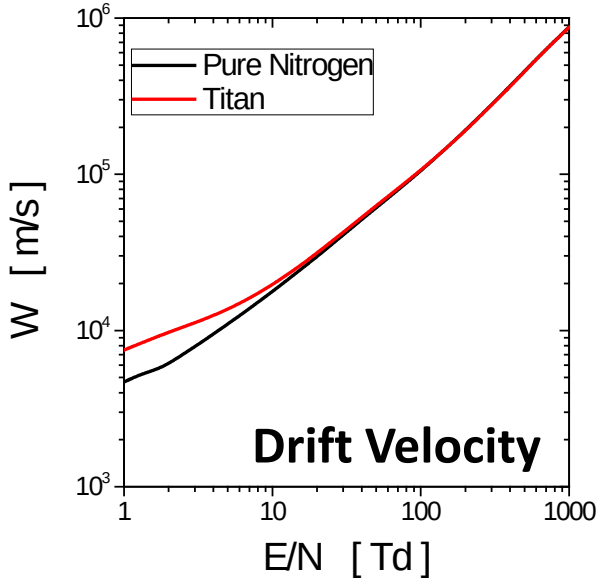
# Electron kinetic processes related to discharges in atmosphere's of Titan

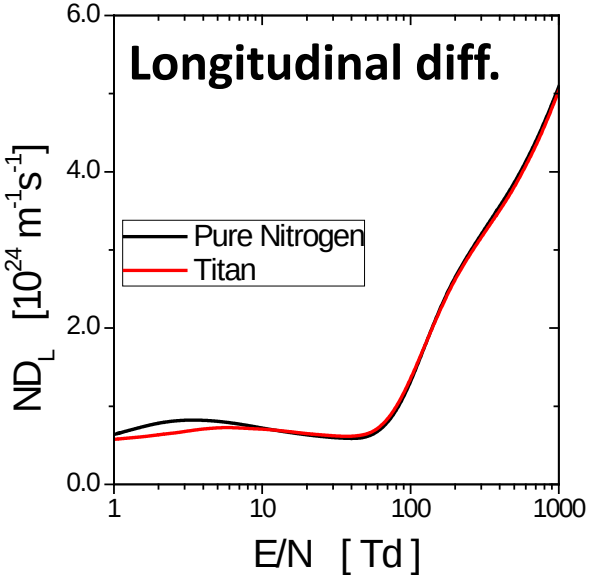


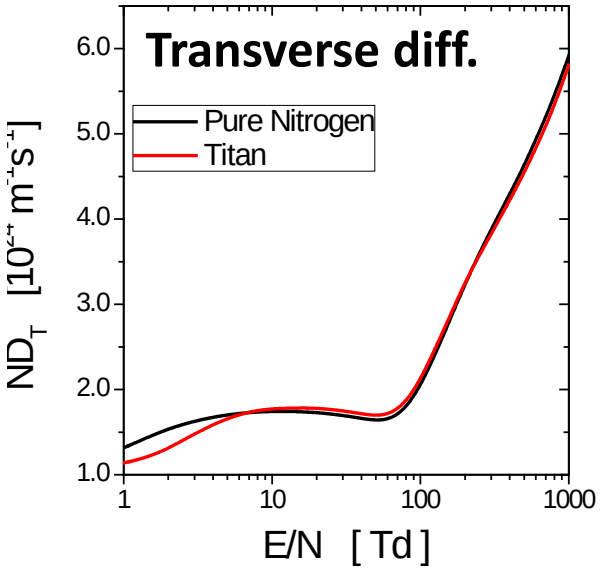


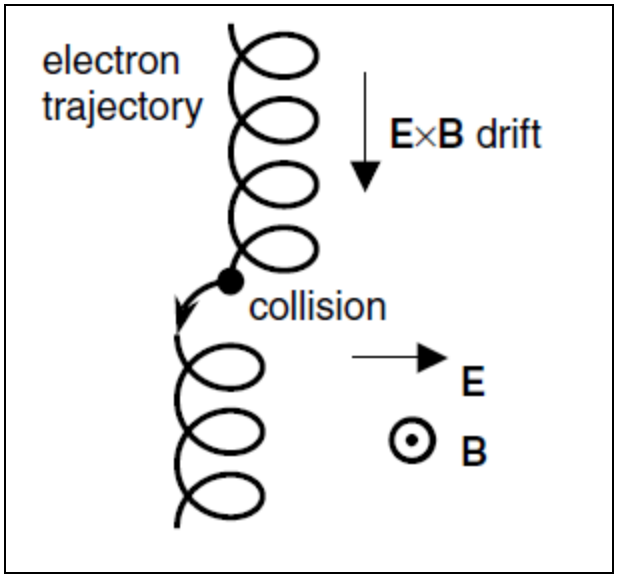



# How do we handle magnetic fields?

# Electron kinetic processes related to magnetized plasma discharges






$\Omega = \frac{eB}{m}$ Cyclotron frequency

$\rho_L = \frac{v_\perp}{\Omega}$ Larmor radius

$v_\perp$ Velocity component perpendicular to B

## Parallel fields:

$$\mathbf{W} = \begin{pmatrix} 0 \\ 0 \\ W_z \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & 0 \\ -D_{xy} & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{pmatrix}$$

## Orthogonal fields:

$$\mathbf{W} = \begin{pmatrix} W_x \\ 0 \\ W_z \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} D_{xx} & 0 & D_{xz} \\ 0 & D_{yy} & 0 \\ D_{zx} & 0 & D_{zz} \end{pmatrix}$$

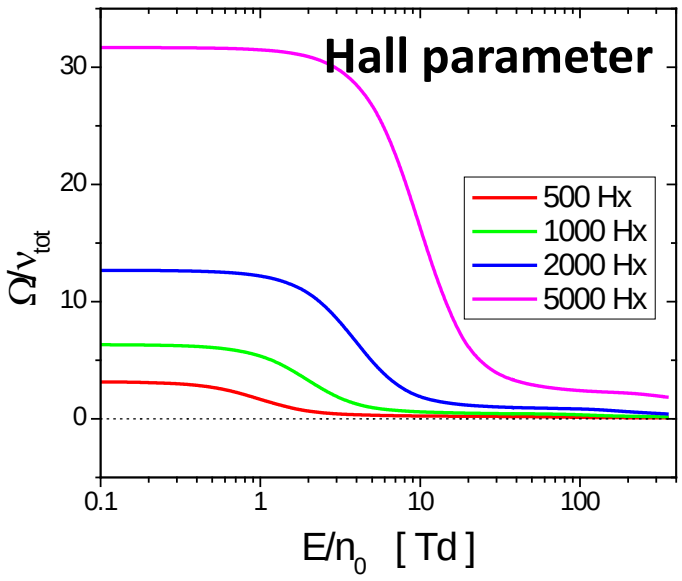
## Arbitrary angle:

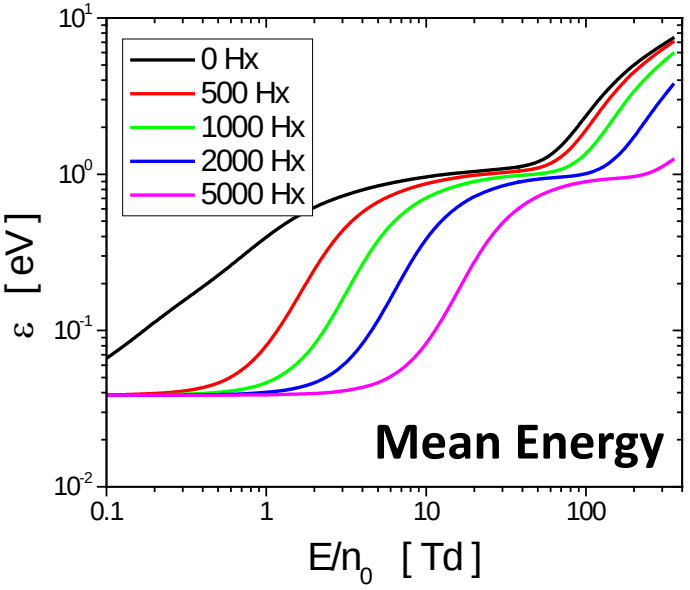
$$\mathbf{W} = \begin{pmatrix} W_x \\ W_y \\ W_z \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

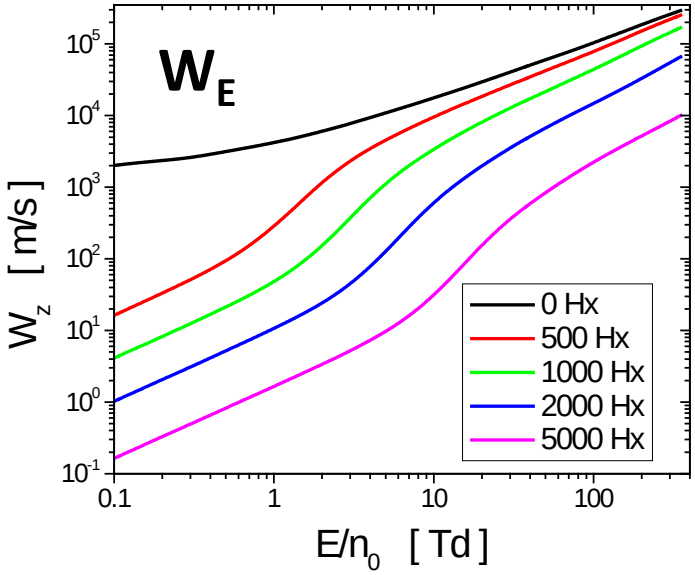
**When magnetic field is present there is a “zoo” of transport coefficients!**

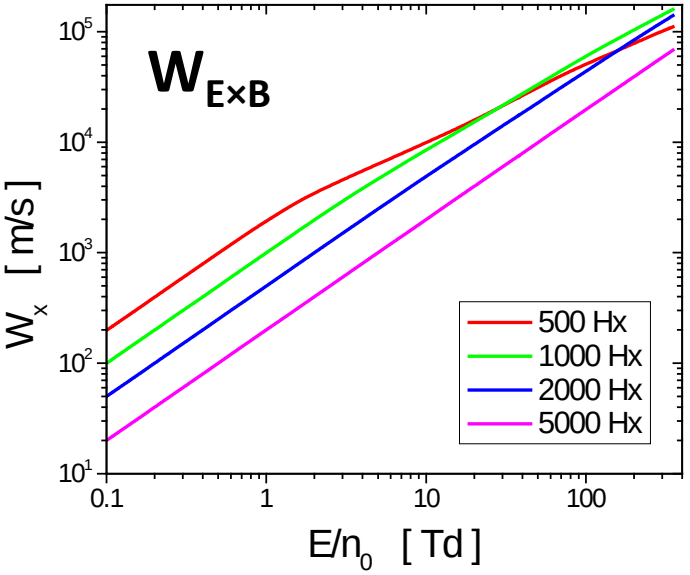
# Electron kinetic processes related to magnetized plasma discharges





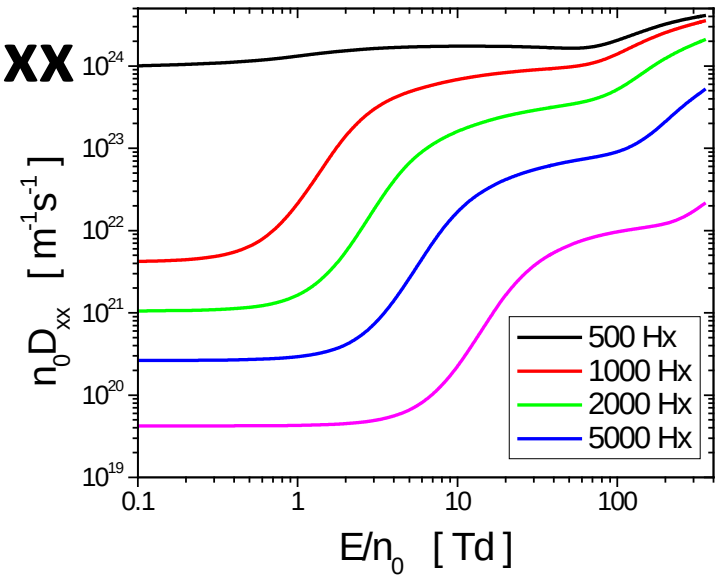



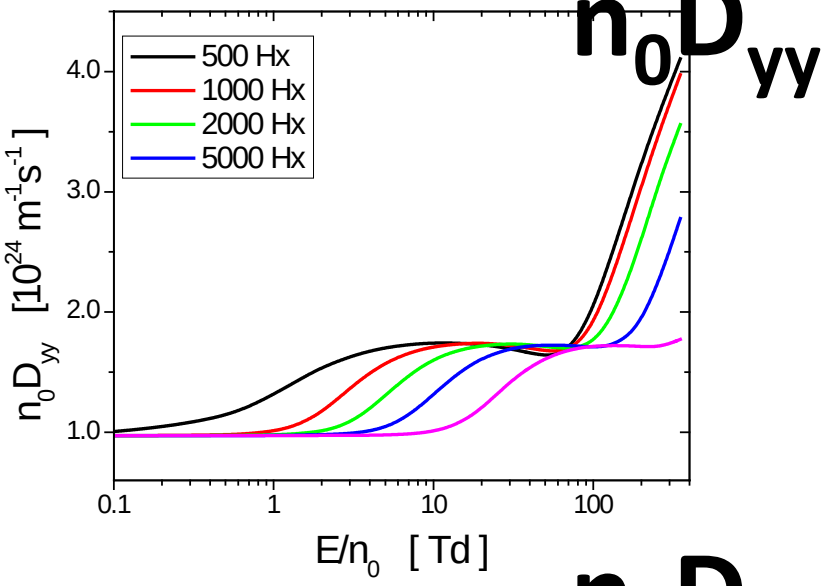
# Electron kinetic processes related to magnetized plasma discharges



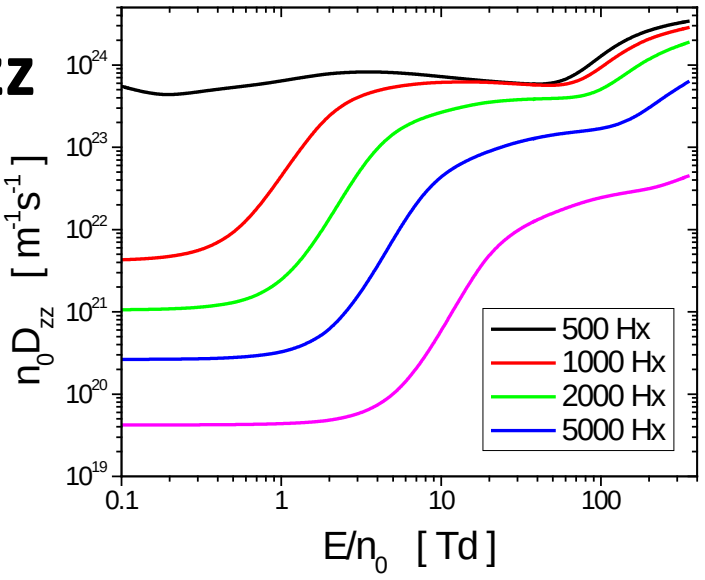
$n_0 D_{xx}$



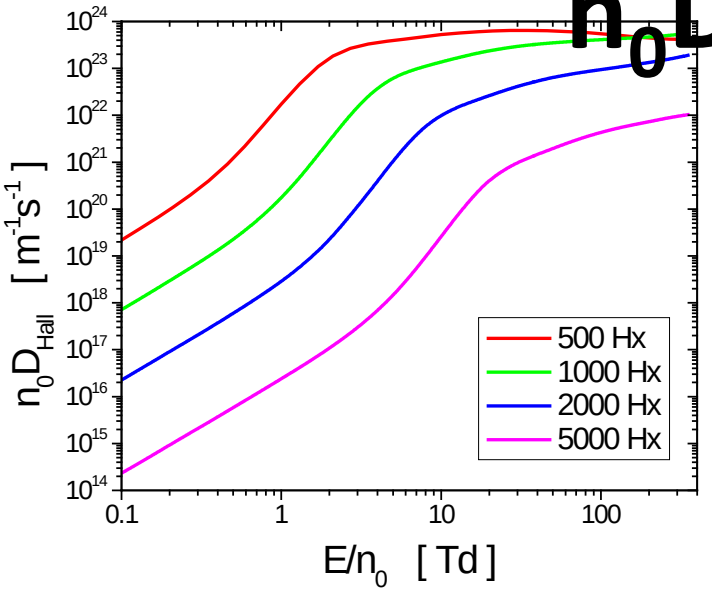

$n_0 D_{yy}$




$n_0 D_{zz}$

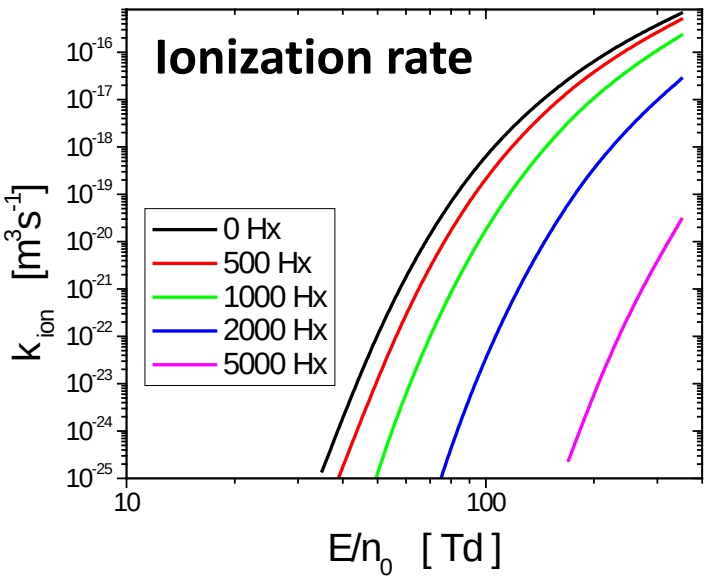


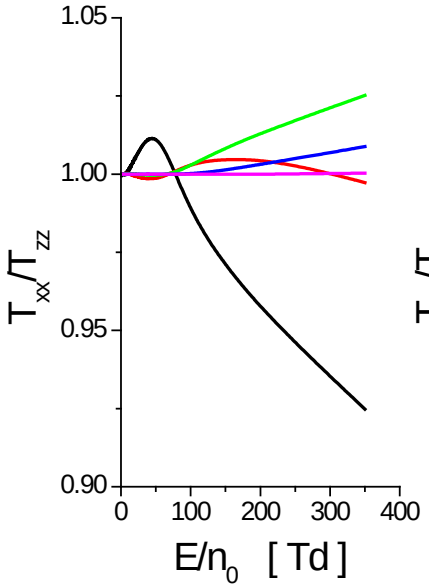

$n_0 D_{Hall}$

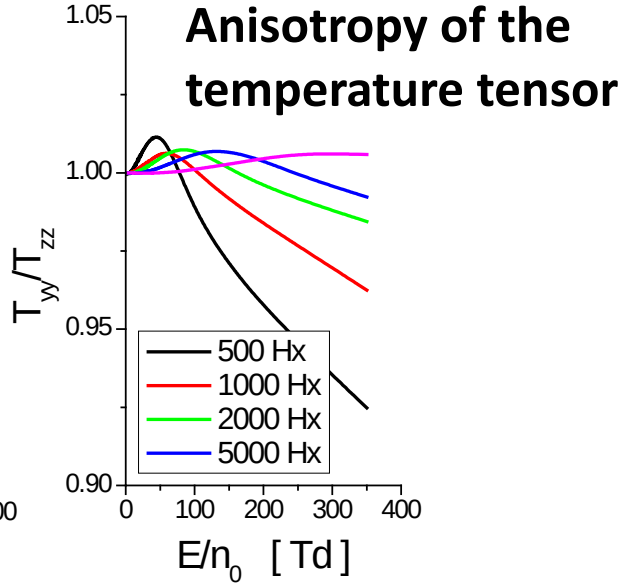


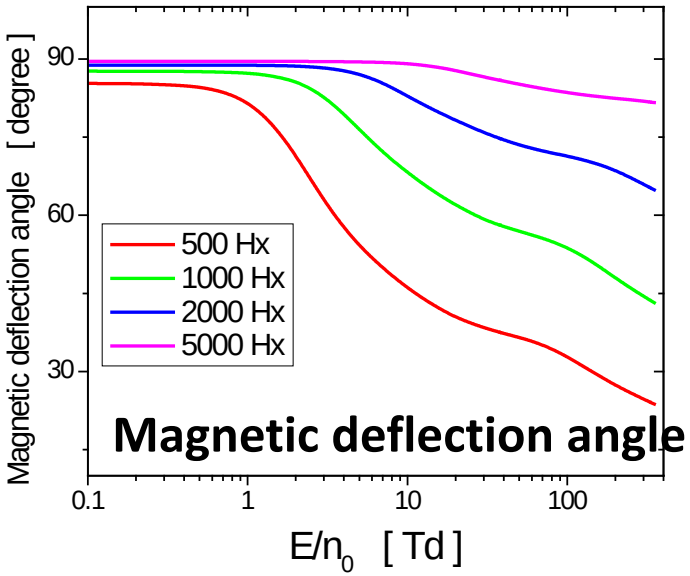
# Electron kinetic processes related to magnetized plasma discharges

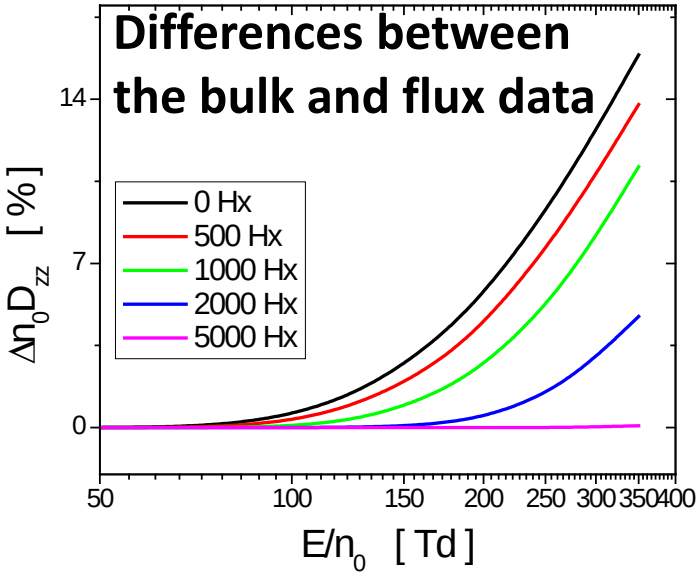




# Electron kinetic processes related to magnetized plasma discharges



| Planet or satellite | Magnetic field in T | Comments |
|---|---|---|
| Mercury | $2\times10^{-7}$ | Not well understood yet. |
| Venus | $<10^{-8}$ | No dynamo at present. |
| Earth | $5\times10^{-5}$ | Core dynamo. |
| Moon | Patchy ($10^{-9}$-$10^{-7}$) | Ancient dynamo? |
| Mars | Patchy ($10^{-9}$-$10^{-4}$) | Ancient dynamo. |
| Jupiter | $4\times10^{-4}$ | Dynamo effect. |
| Io | $<10^{-6}$ ? | Complex. Not well understood. |
| Europa | $10^{-7}$ | Induction response (Salty water ocean). |
| Ganymede | $2\times10^{-6}$ | Dynamo likely. |
| Callisto | $4\times10^{-9}$ | Induction response (Salty water ocean). |
| Saturn | $2\times10^{-5}$ | Dynamo effect. |
| Titan | $<10^{-7}$ | No evidence for a dynamo. |
| Uranus | $2\times10^{-5}$ | Dynamo effect. |
| Neptune | $2\times10^{-5}$ | Dynamo effect. |

# Conclusions and the Future

1. **Swarm physics has a lot to offer to the plasma community in their quest for accuracy.**
2. **We have accurate space- and time-dependent Boltzmann equation solutions in E and B fields (valid for both electrons and ions):**
   - We have examined the effects of three-body attachment on various transport coefficients (attachment cooling and heating);
   - We have observed pressure dependence in transport coefficients;
   - Basic trends in variation of transport data with $E/n_0$ have been investigated for electrons in the atmospheres of Venus and gas giants;
   - Continue to add to the data base of transport for new discharges, stock gases etc.;
   - Further understand non-local behavior.
3. **Full kinetic description of the plasma discharge:**
   - Electron Boltzmann equation;
   - Ion Boltzmann equation;
   - Poisson's equation.