Gaussian Process models for One Hour Ahead prediction of the Dst index

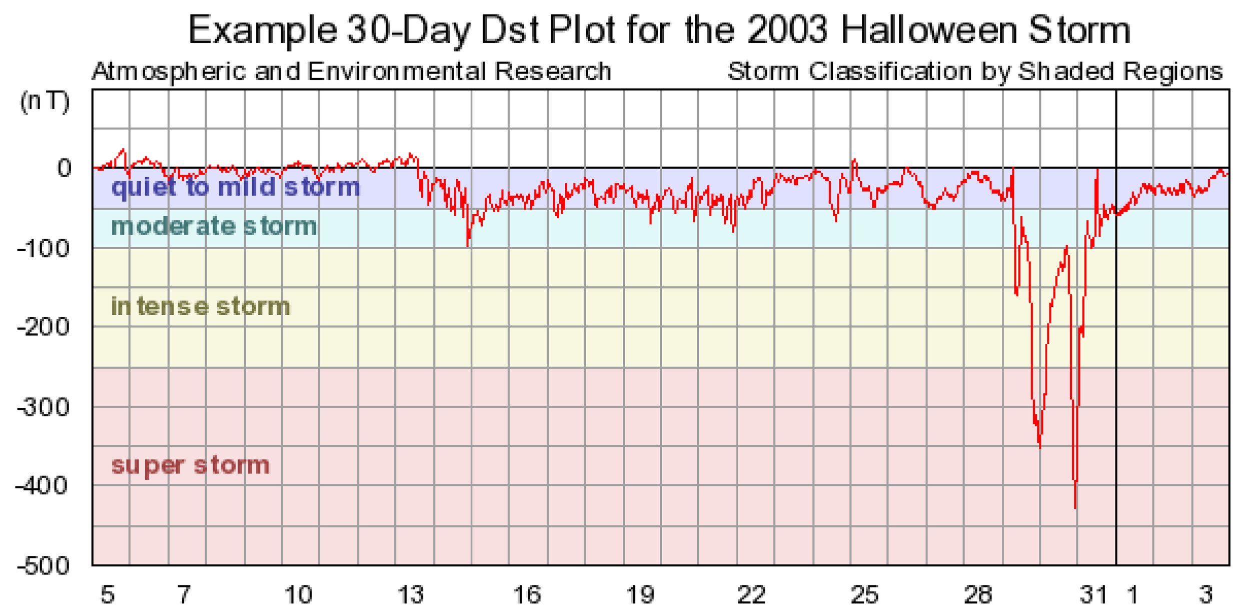
Centrum Wiskunde & Informatica

Mandar Chandorkar[1], Enrico Camporeale[1], Simon Wing[2]

1 Multiscale Dynamics, CWI, Amsterdam, 2 Johns Hopkins University

www.mlspaceweather.org

Geomagnetic Activity and Indexes



Due to the complex nature of geomagnetic response to the solar wind, it is useful to use activity indexes to record and predict the magnetosphere's state.

The Dst index is an index of magnetic activity derived from a network of near-equatorial geomagnetic observatories. Hourly records of Dst are available since 1957. Dst has hourly frequency and and 1 nT resolution.

State of the Art

1. Nonlinear Auto Regressive with exogenous Inputs (solar wind speed, IMF B_z, etc) *NARX/NARMAX*
2. Recurrent Neural Networks
3. Approximate physical models based on ODE: $\frac{dDst(t)}{dt} = Q(t) - \frac{Dst(t)}{\tau}$
4. $\hat{Dst}(t) = Dst(t-1)$: the (in)famous *Persistence* model.

One Step Ahead Dst prediction models compared in Ji et al. (J. Geophys. Res. 2012) over 63 storms (storm-time only)

Model	Reference	Description
TL	Temerin and Li (2002)	Auto regressive model decomposable into additive terms.
NM	Boynton et al. (2011)	Non linear auto regressive with exogenous inputs.
B	Burton et al. (1975)	Prediction of Dst by solving ODE having injection and decay terms.
W	Wang et al. (2003)	Obtained by modification of injection term in Burton.
FL	Fenrich and Luhmann (1998)	Uses polarity of magnetic clouds to predict geomagnetic response.
OM	O'Brien et al. (2000)	Modification of injection term in Burton.

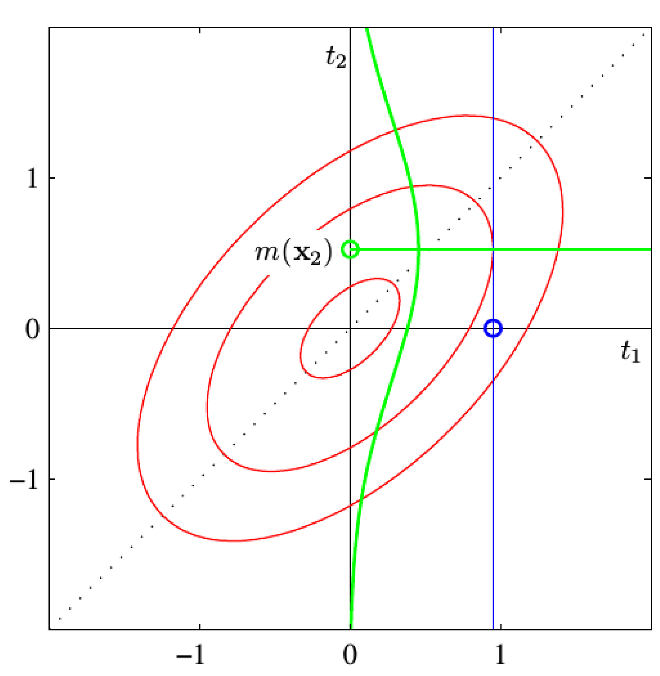
Gaussian Process Regression

Gaussian Process (GP) models specify statistical distributions over functions. In GP models, the finite dimensional distribution of the output data is a multivariate Gaussian specified by equation 3.

$$y = f(x) + \epsilon \tag{1}$$
$$f \sim \mathcal{GP}(m(x), C(x, x')) \tag{2}$$
$$(\mathbf{y} \;\; \mathbf{f}_*)^T \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} K(X,X) + \sigma^2 I & K(X, X_*) \\ K(X_*, X) & K(X_*, X_*) \end{bmatrix}\right) \tag{3}$$

In order to make predictions using GP models, one must calculate the posterior predictive distribution $\mathbf{f}_*|X, \mathbf{y}, X_*$ which is also a multi-variate Gaussian.

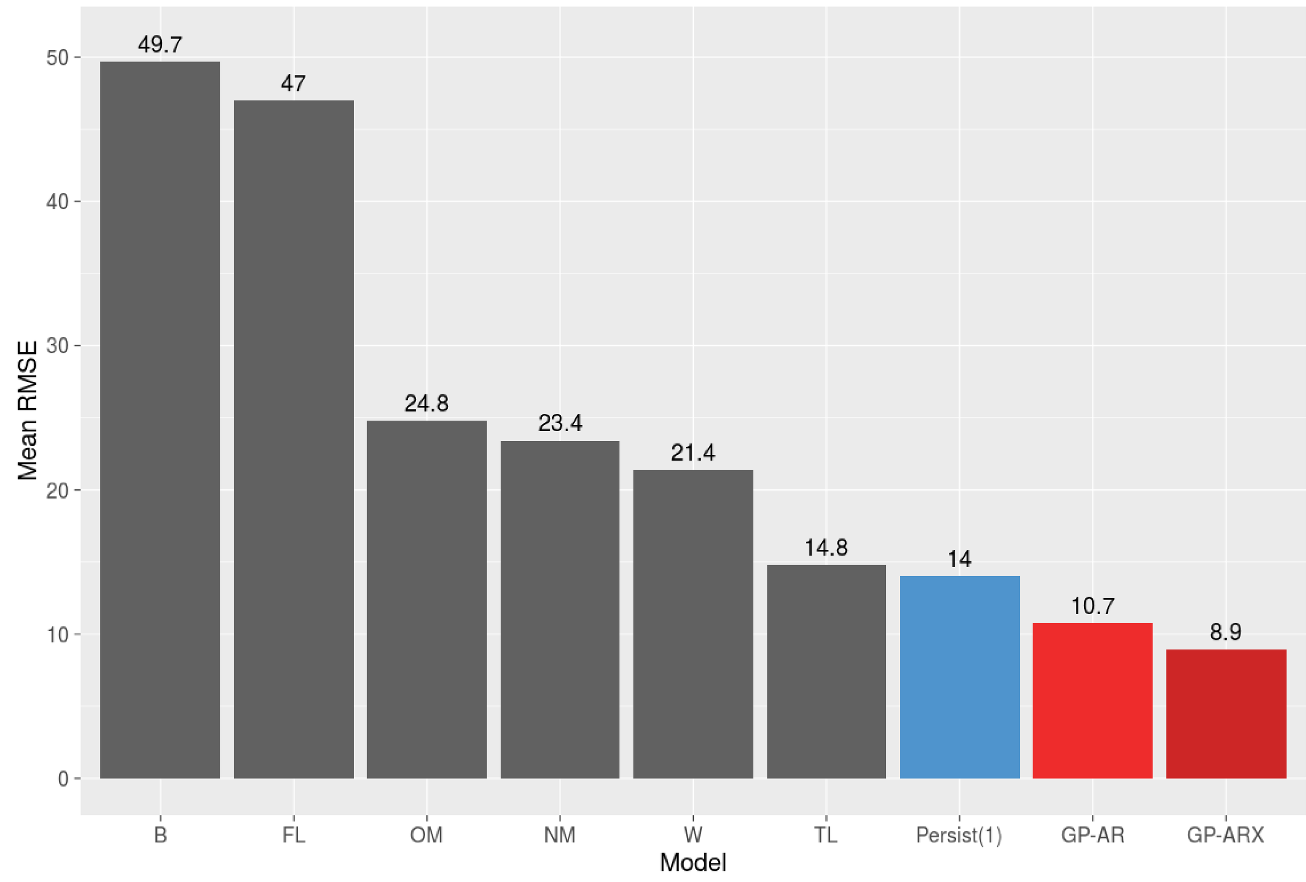
Illustration of the mechanism of Gaussian process regression for the case of one training point and one test point, in which the red ellipses show contours of the joint distribution $p(t_1, t_2)$. Here t_1 is the training data point, and conditioning on the value of t_1, corresponding to the vertical blue line, we obtain $p(t_2|t_1)$ shown as a function of t_2 by the green curve.

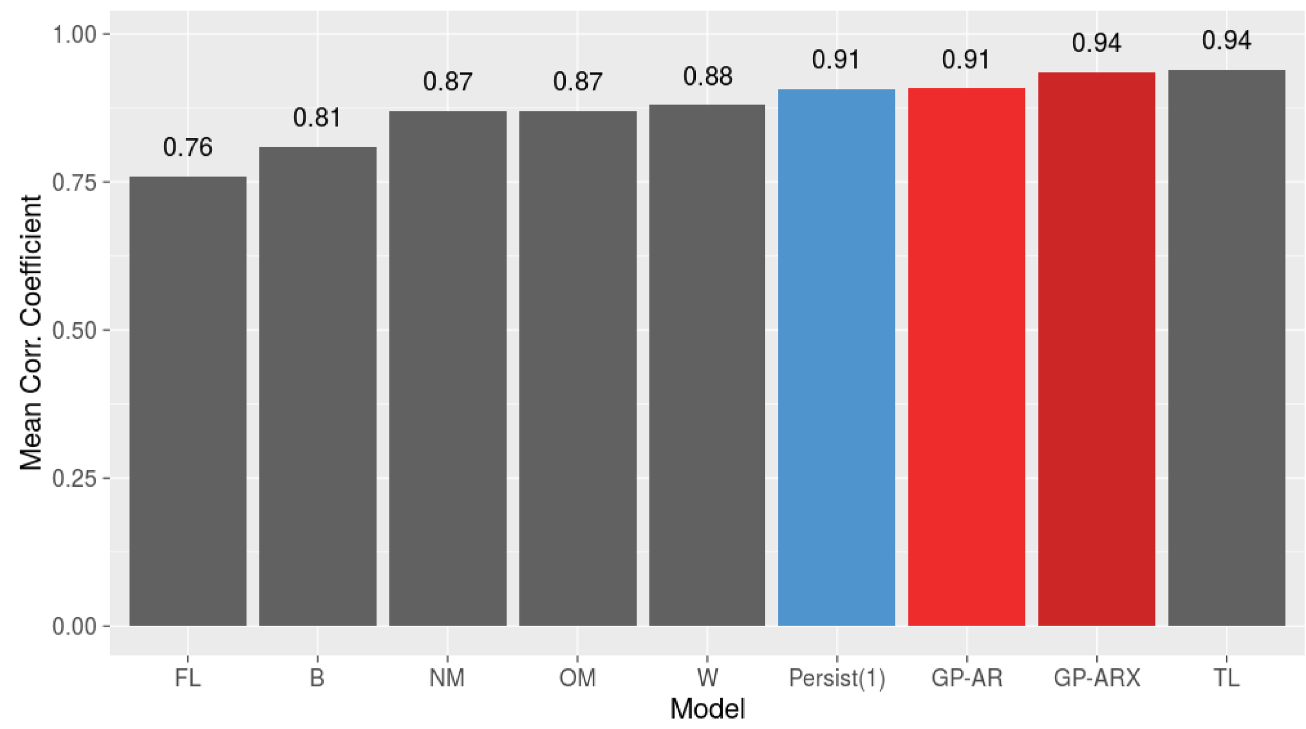


Gaussian Process Dst prediction

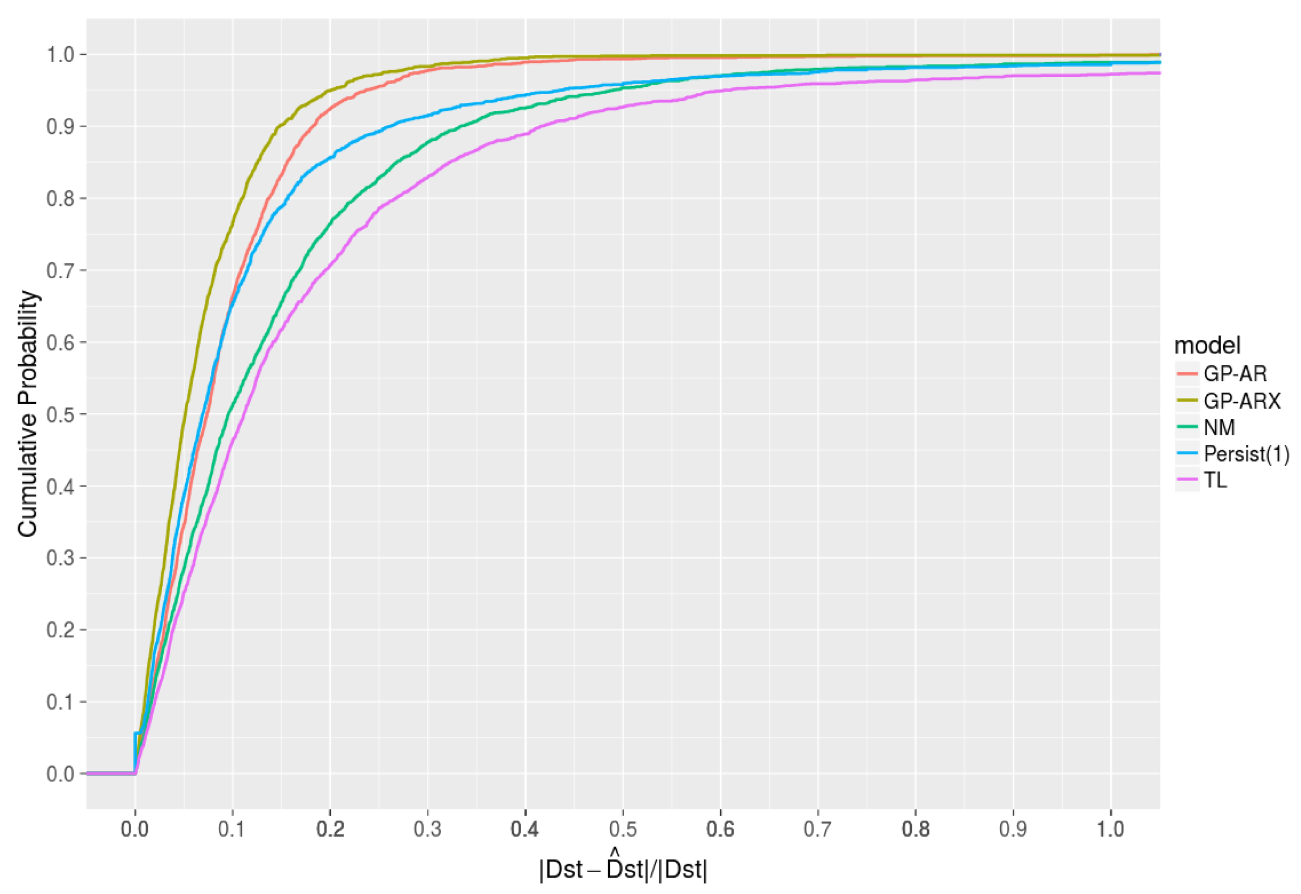
$$Dst(v) \sim \mathcal{GP}(m(v), C(u, v)) \tag{4}$$
$$C_{lin}(u, v) = \mathbb{E}[Dst(u) \times Dst(v)] = u^{\intercal}v + b \tag{5}$$

Using the solar wind speed and z component of IMF as predictive variables, we train a Gaussian Process model to predict one-hour-ahead the Dst index (GP-ARX). We compare the performance of Gaussian Process models with the current state of the art in Dst prediction.





Performance of our models (GP-ARX and GP-AR) compared with previous models: Root Mean Square Error and Mean Correlation Coefficient



Empirical Cumulative Probability of Relative Errors.

Conclusions

1. *Persistence* model must be central in the model evaluation process in the context of *one step ahead* prediction of the Dst index.
2. *Gaussian Process* AR and ARX models give encouraging benefits in One-Step-Ahead prediction even when compared to the *Persistence* model.
3. More informative performance metrics for evaluation and comparison are required.