Galois/Monodromy Groups for Decomposing Minimal Problems in 3D Reconstruction

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Motivation
1. Solve 3D reconstruction task in real-time
2. Efficient solvers for minimal problems [1] are needed
3. Use monodromy to check minimal problems for decomposability

Deck Transformations

\[ f: \frac{\sqrt[n]{x^n + ax^2 + b}}{x} \overset{\sim}{\longrightarrow} \frac{c^2}{x} \quad \text{(unknowns, parameters)} \]
\[ (x, a, b) \rightarrow (a, b) \]

The deck transformation of \( f \) is given by:

\[ \Psi: X \rightarrow X \quad \text{(x, a, b) \rightarrow (-x, a, b)} \]

\( \Psi \) causes \( f \) to decompose:
\[ (x, a, b) \overset{\sim}{\longrightarrow} (x^2, a, b) \overset{\sim}{\longrightarrow} (a, b) \]

Monodromy

\[ \rho: \pi_1(U, z) \rightarrow \text{Sym}(f^{-1}(z)) \]
\[ [\gamma] \mapsto \sigma_\gamma \]

Monodromy group \( \text{Mon}(f) \) is defined as the image of \( \rho \):

\[ \text{Deck}(f) \cong \text{Cent}_{\mathbb{Z}_2}(\text{Mon}(f)) \]

The decomposability is caused by the non-triviality of the group of deck transformations:

\[ \text{Deck}(f) = (\Psi_1, \Psi_2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \]

\( \Psi_1 \) reflects and rotates the \( i \)-th camera

Classical 5-point Minimal Problem

The 5-point problem decomposes:

\( (R, t, \alpha_1, \ldots, \beta_5, p) \overset{\sim}{\longrightarrow} [k] \times R \times p \overset{10:1}{\longrightarrow} p \)

where \( p = (x_1, \ldots, y_5) \). This decomposition is caused by the presence of deck transformation:

\[ R \gamma R = R, \quad \det R = 1, \quad \beta y_i = R \alpha x_i + t, \quad \forall i = 1, \ldots, 5. \]

Results, Finding Decomposable Minimal Problems

The decomposability is caused by the non-triviality of the group of deck transformations:

\[ \text{Deck}(f) = (\Psi_1, \Psi_2) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \]

\( \Psi_1 \) reflects and rotates the \( i \)-th camera

References