GALOIS/MONODROMY GROUPS FOR DECOMPOSING MINIMAL PROBLEMS IN 3D RECONSTRUCTION

Timothy Duff[1], Viktor Korotynskiy[2], Tomas Pajdla[2], Margaret H. Regan[3]

[1]University of Washington, [2]CIIRC CTU, [3]Duke University

MOTIVATION

1. Solve 3D reconstruction task in real-time
2. Efficient solvers for minimal problems [1] are needed
3. Use monodromy to check minimal problems for decomposability

DECK TRANSFORMATIONS

$$f:\ \underbrace{\mathbf{V}_{\mathbb{C}}(x^4+ax^2+b)}_{X} \xrightarrow{4:1} \underbrace{\mathbb{C}^2}_{Z}$$

$$(x,a,b) \longmapsto (a,b)$$

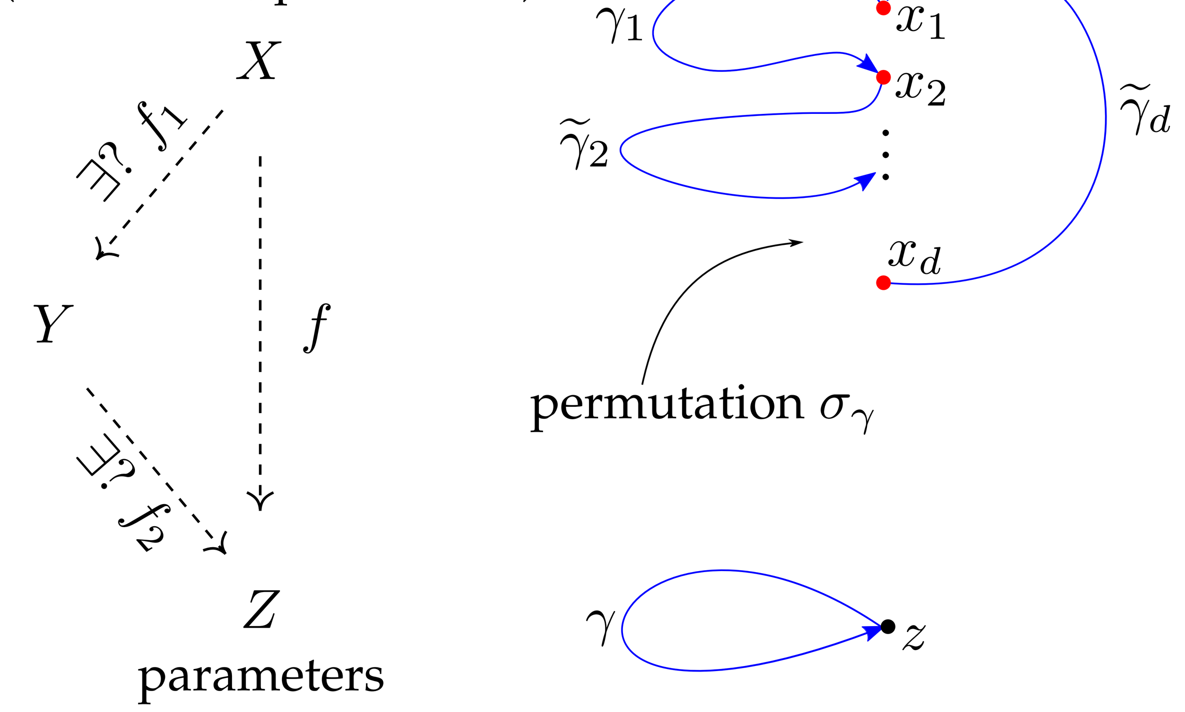
The deck transformation of f is given by

$$\Psi\colon X \to X$$
$$(x,a,b) \mapsto (-x,a,b)$$

Ψ causes f to decompose:

$$(x,a,b) \xrightarrow{2:1} (x^2,a,b) \xrightarrow{2:1} (a,b)$$

MONODROMY



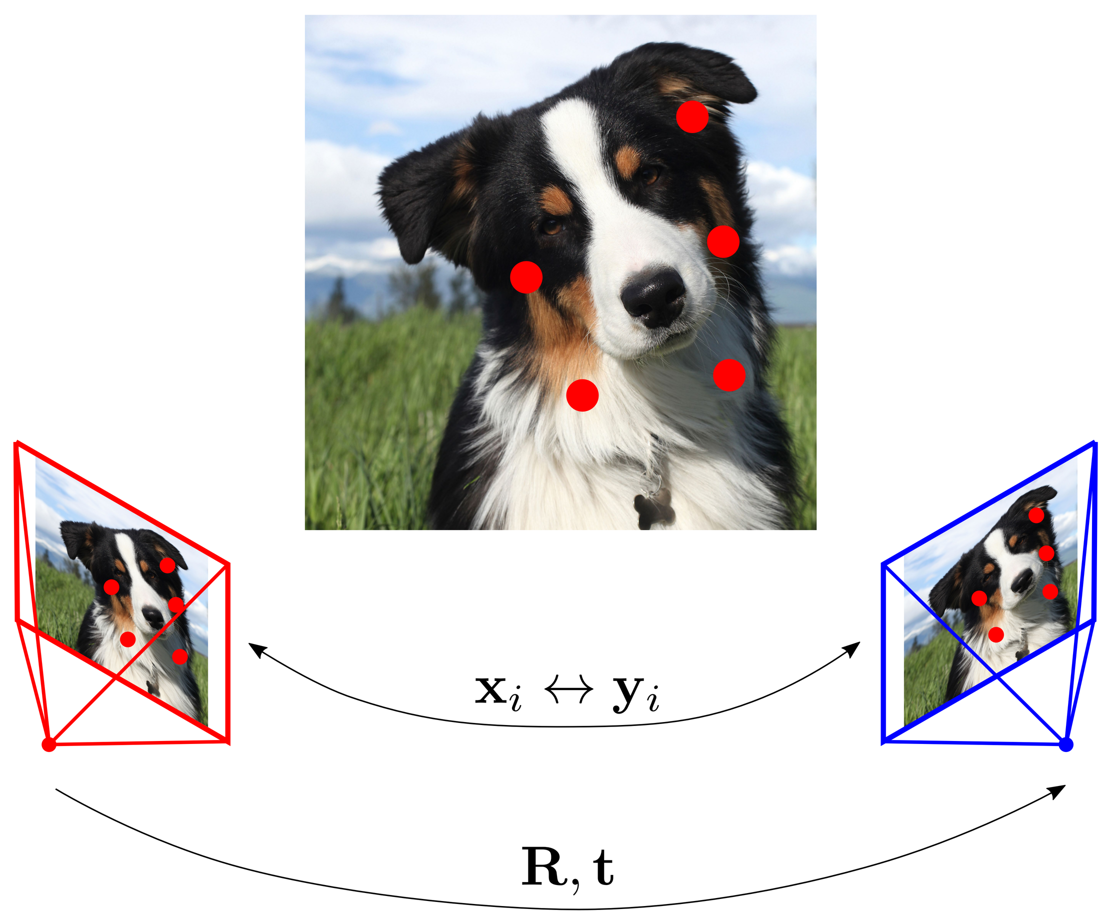
$$\rho : \pi_1(U,z) \to Sym(f^{-1}(z))$$
$$[\gamma] \mapsto \sigma_\gamma$$

$$\mathrm{Mon}(f) \stackrel{\text{def}}{=} \mathrm{im}(\rho)$$

$\mathrm{Mon}(f)$ is imprimitive $\iff$ f is decomposable

$$\mathrm{Deck}(f) \cong \mathrm{Cent}_{S_d}(\mathrm{Mon}(f))$$

CLASSICAL 5-POINT MINIMAL PROBLEM

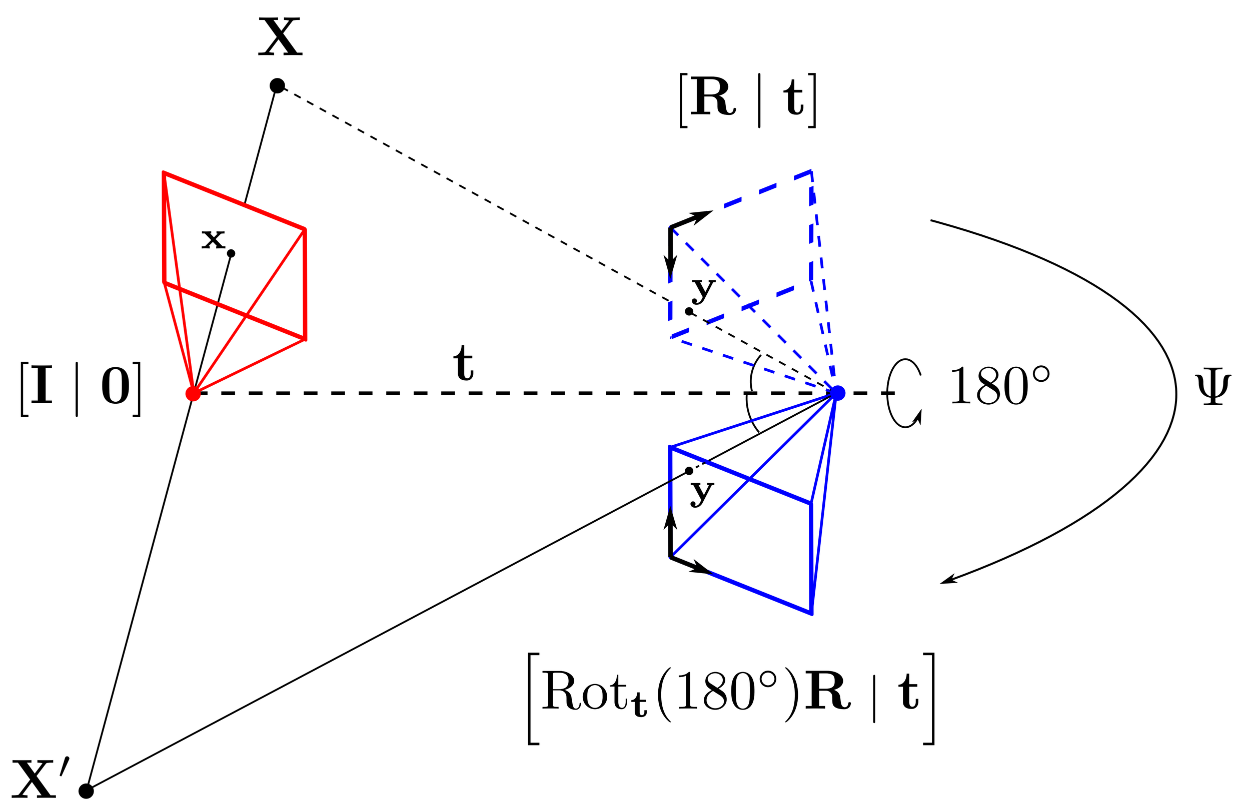


$$\mathbf{R}^\top\mathbf{R} = \mathbf{I}, \quad \det \mathbf{R} = 1,$$
$$\beta_i\mathbf{y}_i = \mathbf{R}\alpha_i\mathbf{x}_i + \mathbf{t}, \quad \forall\, i = 1,\ldots,5.$$

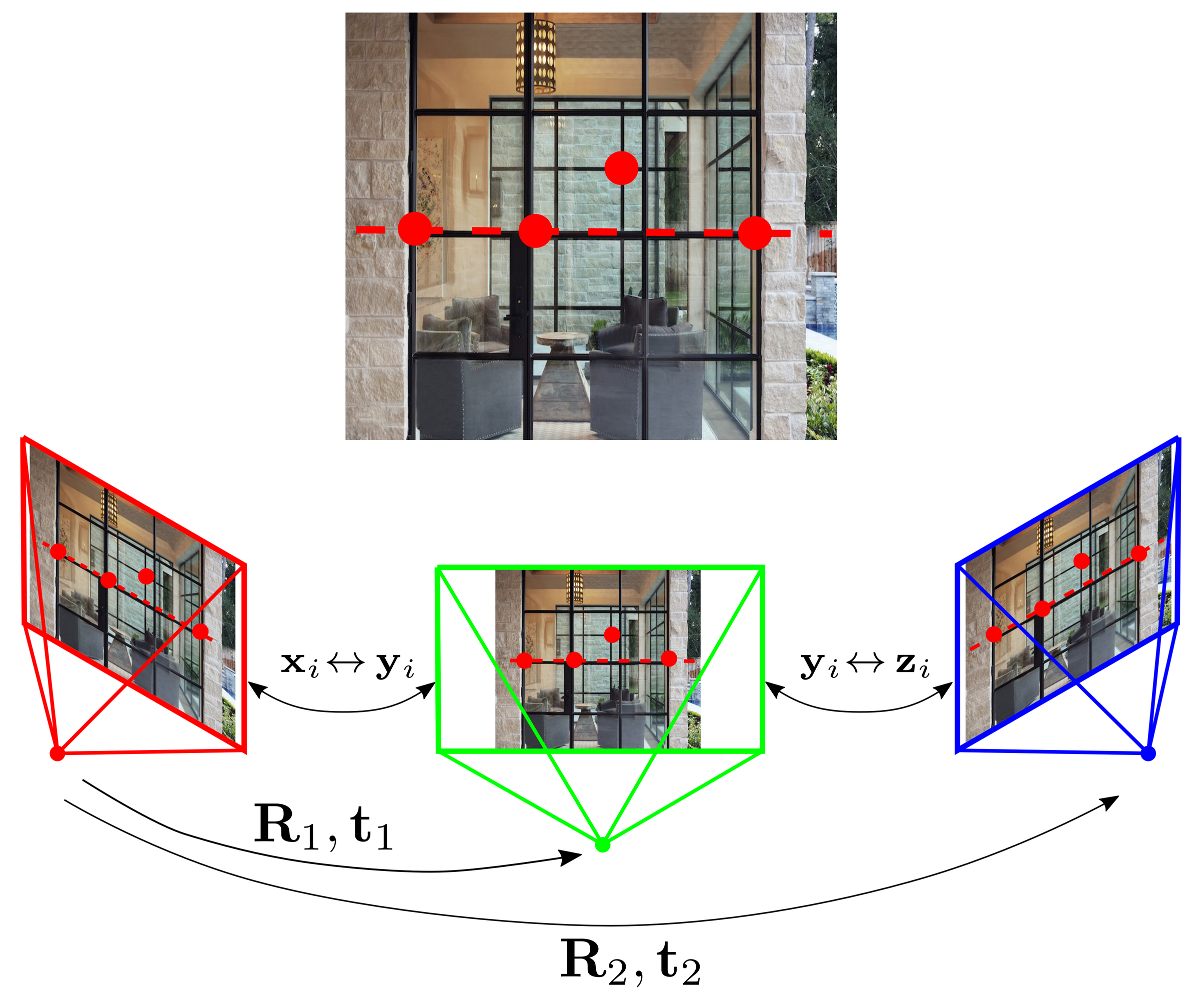
The 5-point problem decomposes:

$$(\mathbf{R},\mathbf{t},\alpha_1,\ldots,\beta_5,\mathbf{p}) \xrightarrow{2:1} ([\mathbf{t}]_\times\mathbf{R},\mathbf{p}) \xrightarrow{10:1} \mathbf{p}$$

where $\mathbf{p} = (\mathbf{x}_1,\ldots,\mathbf{y}_5)$. This decomposition is caused by the presence of deck transformation:



RESULTS, FINDING DECOMPOSABLE MINIMAL PROBLEMS



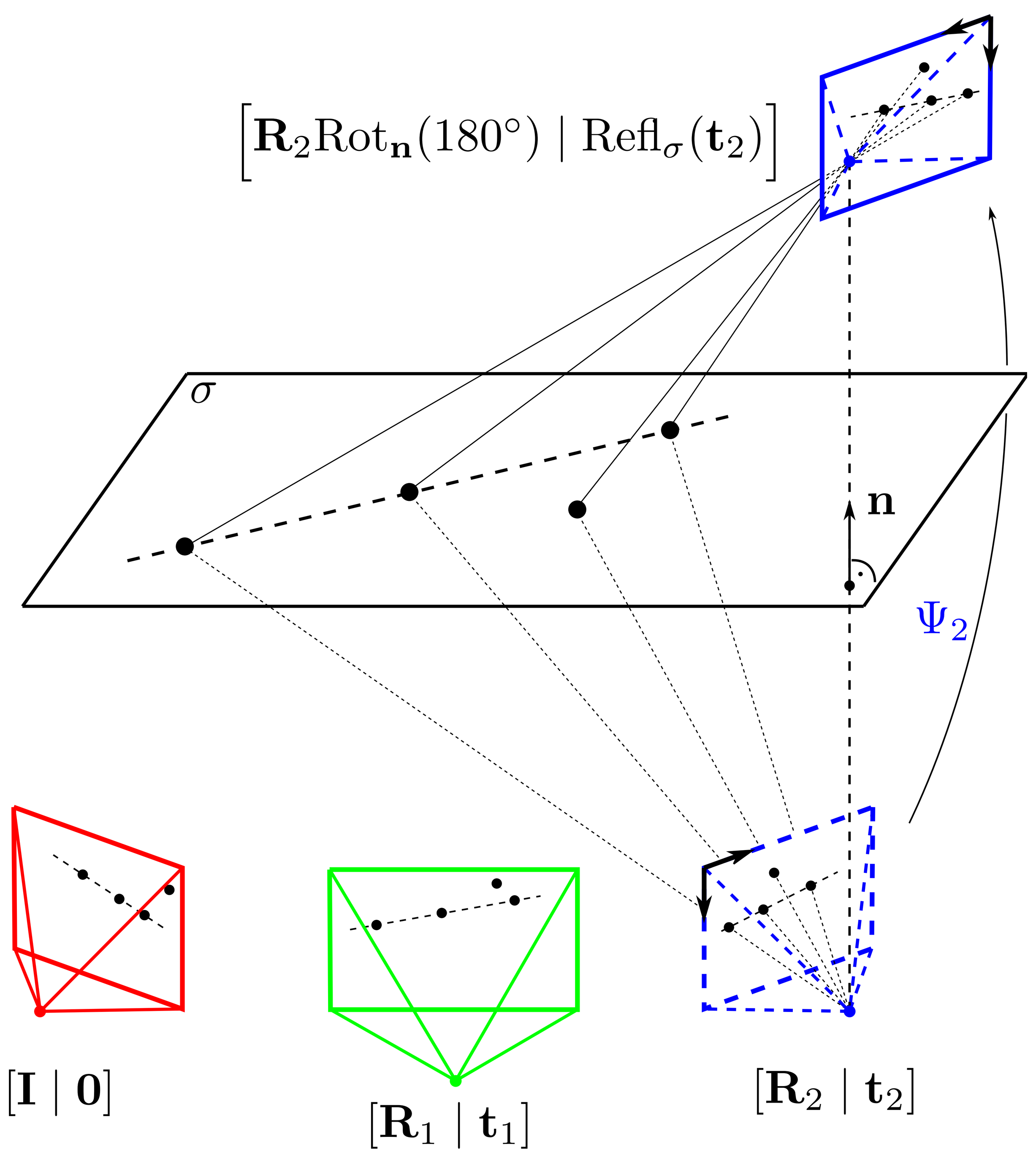
The decomposability is caused by the non-triviality of the group of deck transformations:

$$\mathrm{Deck}(\,\cdot\,) = \langle \Psi_1, \Psi_2 \rangle \cong \mathbb{Z}_2 \times \mathbb{Z}_2$$

Ψ_i reflects and rotates the i-th camera

$$X \xrightarrow{4:1} Y \xrightarrow{16:1} Z$$

$$\begin{bmatrix}\mathbf{H}_1\\ \mathbf{H}_2\\ \mathbf{x}_1\\ \vdots\\ \mathbf{z}_4\end{bmatrix} \longmapsto \begin{bmatrix} s_1 = \frac{1}{\mathbf{H}^2_{1,(1,1)}}\\ \mathbf{S}_1 = \frac{1}{\mathbf{H}_{1,(1,1)}}\mathbf{H}_1\\ s_2 = \frac{1}{\mathbf{H}^2_{2,(1,1)}}\\ \mathbf{S}_2 = \frac{1}{\mathbf{H}_{2,(1,1)}}\mathbf{H}_2\\ \mathbf{x}_1\\ \vdots\\ \mathbf{z}_4\end{bmatrix} \longmapsto \begin{bmatrix}\mathbf{x}_1\\ \vdots\\ \mathbf{z}_4\end{bmatrix}$$



FUTURE RESEARCH

We plan to design the extension of the algorithm from [2] that will compute deck transformations by rational function interpolation using data obtained from monodromy computations.

We would also like to investigate if the elimination templates [3] generated from the revealed decompositions are "smaller" than the ones generated from the original formulation.

REFERENCES

[1] T. Duff, K. Kohn, A. Leykin, and T. Pajdla. Point-line minimal problems in complete multi-view visibility, 2019.

[2] V. Larsson and K. Åström. Uncovering symmetries in polynomial systems, 2016.

[3] E. Martyushev, J. Vrablikova, and T. Pajdla. Optimizing elimination templates by greedy parameter search, 2022.

This work was partly supported by NSF Mathematical Sciences Postdoctoral Research Fellowship (DMS-2103310), NSF DMS #1719968 and #2001267, Schmitt Leadership Fellowship in Science and Engineering and NSF grant CCF-1812746.

This work was partly supported by European Regional Development Fund under the project IMPACT (reg. no. CZ.02.1.01/0.0/0.0/15 003/0000468), the EU H2020 ARtwin No. 856994 projects and Grant Agency of the CTU project SGS21/056/OHK3/1T/13.