Robustness of the Maximal Covering Location Problem

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Emergency Medical Service

The organisation and coordination of out-of-hospital:

- Acute medical care
- Transportation of patients
Emergency Medical Service

The organisation and coordination of out-of-hospital:
  ▶ Acute medical care
  ▶ Transportation of patients

Service providers are responsible for:
  ▶ Handling 112 emergency medical calls
  ▶ Dispatching of ambulances
Emergency Medical Service

The organisation and coordination of out-of-hospital:

- Acute medical care
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Situation in The Netherlands:
Emergency Medical Service

Situation in The Netherlands:

- 24 regional services
Emergency Medical Service

Situation in The Netherlands:

- 24 regional services
- 200 ambulance bases

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Emergency Medical Service

Situation in The Netherlands:

- 24 regional services
- 200 ambulance bases
- 700 ambulances
Emergency Medical Service

Situation in The Netherlands:

- 24 regional services
- 200 ambulance bases
- 700 ambulances
- 1.1 million trips per year
Emergency Medical Service

Situation in The Netherlands:

- 24 regional services
- 200 ambulance bases
- 700 ambulances
- 1.1 million trips per year

Services are tasked to optimise their performance
Placement of Ambulance Bases
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Use optimisation models to determine optimal base locations
Placement of Ambulance Bases

Use optimisation models to determine optimal base locations

- Facility location model
Placement of Ambulance Bases

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- Facility location model
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Objective

$$\max \sum_{j \in J} d_j z_j$$

Constraints

$$\sum_{i \in I} x_i = p$$

$$\sum_{i \in I} a_{ij} x_i \geq z_j \quad \forall j \in J$$

$$x_i \in \mathbb{B} \quad \forall i \in I$$

$$z_j \in \mathbb{B} \quad \forall j \in J$$

Sets:
- Possible base locations $I$
- Demand points $J$

Parameters:
- Demand weights $d_j \in \mathbb{R} \geq 0$
- Number of bases $p \in \mathbb{N}$
- Adjacency $a_{ij} \in \mathbb{B}$

Variables:
- Opened bases $x_i$
- Covered points $z_j$
Placement of Ambulance Bases

Objective

\[ \max \sum_{j \in J} d_j z_j \]

Constraints

\[ \sum_{i \in I} x_i = p \]
\[ \sum_{i \in I} a_{ij} x_i \geq z_j \quad \forall j \in J \]
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Sets:

- Possible base locations $\mathcal{I}$
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Placement of Ambulance Bases

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Sets:
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Parameters:
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Placement of Ambulance Bases

### Objective

\[
\max \sum_{j \in \mathcal{J}} d_j z_j
\]

### Constraints

\[
\sum_{i \in \mathcal{I}} x_i = p
\]

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\sum_{i \in \mathcal{I}} a_{ij} x_i \geq z_j \quad \forall j \in \mathcal{J}
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Sets:
- Possible base locations \( \mathcal{I} \)
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Variables:
- Opened bases \( x_i \)
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Model Robustness
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Type of robustness:
Model Robustness

Type of robustness:

▶ Model parameters
Model Robustness

Type of robustness:

▶ Model parameters
▶ Related to data uncertainty
Model Robustness

Type of robustness:
- Model parameters
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Robustness:
- Solutions are insensitive to (small) parameter changes
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Robustness:
- Solutions are insensitive to (small) parameter changes
- Results are more reliable
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Type of robustness:
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Focus on demand weights $d_j$
Model Robustness

Suppose:

- Up to 5% deviation in demand $d_j$ is possible
- Total demand is fixed
Model Robustness

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Where to place ambulance bases?
Model Robustness

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Where to place ambulance bases?

Safe approach:
Model Robustness

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Where to place ambulance bases?

Safe approach:

- Best coverage in case of worst-case realisation of demand
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Where to place ambulance bases?

Safe approach:
- Best coverage in case of worst-case realisation of demand
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Suppose:

▶ Up to 5% deviation in demand $d_j$ is possible
▶ Total demand is fixed

Where to place ambulance bases?

Safe approach:

▶ Best coverage in case of worst-case realisation of demand
▶ Worst-case realisation depends on base locations!
Model Robustness

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- Up to 5% deviation in demand $d_j$ is possible
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Use Robust Optimisation techniques
Model Robustness

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Where to place ambulance bases?

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- Best coverage in case of worst-case realisation of demand
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Use Robust Optimisation techniques
- Uses duality theory
Model Robustness

Suppose:

- Up to 5% deviation in demand is possible: \( d_j \in [d_j, \bar{d}_j] \)
- Total demand is fixed to \( \Delta \)
Model Robustness

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New variables: \( u, v_j^+, v_j^- \ \in [0, 1] \)
Model Robustness

Suppose:

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- Total demand is fixed to \( \Delta \)

New variables: \( u, v_j^+, v_j^- \in [0, 1] \)

Robust objective:

\[
\Delta u \overset{\text{estimated total coverage}}{=} - \sum_{j \in \mathcal{J}} \bar{d}_j v_j^+ + \sum_{j \in \mathcal{J}} d_j v_j^-
\]

\( \text{correction overestimates} \quad \text{correction underestimates} \)
Model Robustness

Suppose:
- Up to 5% deviation in demand is possible: \(d_j \in [\underline{d}_j, \overline{d}_j]\)
- Total demand is fixed to \(\Delta\)

New variables: \(u, v_j^+, v_j^- \in [0, 1]\)

Robust objective:

\[
\begin{align*}
\Delta u & \geq \text{estimated total coverage} \ - \sum_{j \in \mathcal{J}} \overline{d}_j v_j^+ \ + \sum_{j \in \mathcal{J}} \underline{d}_j v_j^- \\
& \text{correction overestimates} \quad \text{correction underestimates}
\end{align*}
\]

Also some additional constraints
Model Robustness

Suppose:

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- Total demand is fixed
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We have two models:
- Normal coverage model
- Robust coverage model
Model Robustness

Suppose:
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We have two models:
- Normal coverage model
- Robust coverage model
- Both are equivalent under these assumptions!
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Suppose:

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We have two models:

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The model is robust with respect to demand uncertainty
Model Robustness

Alternative parameter uncertainty:
Model Robustness

Alternative parameter uncertainty:

- Total demand is fixed
Model Robustness

Alternative parameter uncertainty:

- Total demand is fixed
- Estimated demand $\hat{d}_j$ is the rate of Poisson process
Model Robustness

Alternative parameter uncertainty:
▶ Total demand is fixed
▶ Estimated demand $\hat{d}_j$ is the rate of Poisson process
▶ Uncertainty up to a standard deviation in demand
Model Robustness

Alternative parameter uncertainty:

- Total demand is fixed
- Estimated demand $\hat{d}_j$ is the rate of Poisson process
- Uncertainty up to a standard deviation in demand

Same robust coverage model, but with:

$$d_j \in [\underline{d}_j, \overline{d}_j] = \left[\hat{d}_j - \sqrt{\hat{d}_j}, \hat{d}_j + \sqrt{\hat{d}_j}\right]$$
Model Robustness

Alternative parameter uncertainty:

- Total demand is fixed
- Estimated demand \( \hat{d}_j \) is the rate of Poisson process
- Uncertainty up to a standard deviation in demand

Two models:

- Normal and robust coverage models
Model Robustness

Alternative parameter uncertainty:

- Total demand is fixed
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Two models:

- Normal and robust coverage models
- Not equivalent under these assumptions!
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Numerical results:

Solution differs in 32 of 609 cases (5%)
Coverage difference is very small (<1%)
Model Robustness

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Conclusion
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Maximal Covering Location problem (MCLP):
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Maximal Covering Location problem (MCLP):
▶ Coverage optimisation model
Conclusion

Maximal Covering Location problem (MCLP):

▶ Coverage optimisation model
▶ Robust for two common types of demand uncertainty

Robust Optimisation:

▶ General optimisation technique
▶ Useful for worst-case robust solutions

See also:

Conclusion

Maximal Covering Location problem (MCLP):

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▶ Robust for two common types of demand uncertainty

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Conclusion

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- Robust for two common types of demand uncertainty

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- General optimisation technique
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- See also:

Questions?