Where is the Research on Evolutionary Multi-objective Optimization Heading To?

Carlos A. Coello Coello

CINVESTAV-IPN
Evolutionary Computation Group (EVOCINV)
Departamento de Computación
Av. IPN No. 2508, Col. San Pedro Zacatenco
México, D.F. 07360, MEXICO
ccoello@cs.cinvestav.mx

JoLEA, June 2023

Outline

Motivation

Most problems in nature have several (possibly conflicting) objectives to be satisfied (e.g., design a bridge for which want to minimize its weight and cost while maximizing its safety). Many of these problems are frequently treated as single-objective optimization problems by transforming all but one objective into constraints.

Formal Definition

Find the vector $\vec{x}^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$ which will satisfy the m inequality constraints:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{1}$$

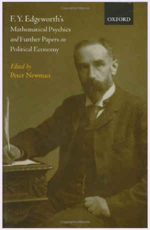
the p equality constraints

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{2}$$

and will optimize the vector function

$$\vec{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]^T \tag{3}$$

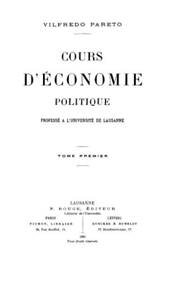
Notion of Optimality in MOPs



Having several objective functions, the notion of “optimum” changes, because in MOPs, the aim is to find good compromises (or “trade-offs”) rather than a single solution as in global optimization.

The notion of “optimum” that is most commonly adopted is that originally proposed by Francis Ysidro Edgeworth (in 1881) in his book entitled **Mathematical Psychics**.

Notion of Optimality in MOPs



VILFREDO PARETO

COURS
D'ÉCONOMIE
POLITIQUE

PROFESSÉ A L'UNIVERSITÉ DE LAUSANNE

TOME PREMIER

LAUSANNE
F. ROUGE, ÉDITEUR
Librairie de l'Université.

PARIS
PICHON, LIBRAIRE
24, Rue Soufflot, 24.

LEIPZIG
DUNCKER & HUMBLOT
17, Dresdnerstrasse, 17.

1896
Tous droits réservés.

This notion was generalized by the italian economist Vilfredo Pareto (in 1896) in his book **Cours d'Economie Politique**. Although some authors call *Edgeworth-Pareto optimum* to this notion (originally called **ophelimity**) it is normally preferred to use the most commonly accepted term: **Pareto optimum**.

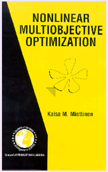
Notion of Optimality in MOPs

Pareto Optimality

We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ is **Pareto optimal** if there does not exist another $\vec{x} \in \mathcal{F}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j (assuming that all the objectives are being minimized).

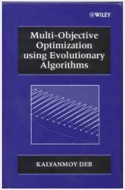
In words, this definition says that $\vec{x}^*$ is **Pareto optimal** if there exists no feasible vector of decision variables $\vec{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. This concept normally produces a set of solutions called the **Pareto optimal set**. The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called **nondominated**. The image of the Pareto optimal set is called the **Pareto front**.

Mathematical Programming Techniques



Currently, there are over 30 mathematical programming techniques for multiobjective optimization. However, these techniques tend to generate elements of the Pareto optimal set one at a time. Additionally, most of them are very sensitive to the shape of the Pareto front (e.g., they do not work when the Pareto front is concave or when the front is disconnected).

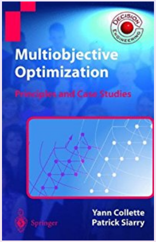
A Taxonomy of MOEAs



The Old Days

- Non-Elitist Non-Pareto-based Methods
  - Lexicographic Ordering
  - Linear Aggregating Functions
  - VEGA
  - ε-Constraint Method
  - Target Vector Approaches

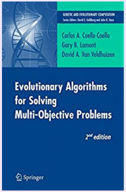
A Taxonomy of MOEAs



The Old Days

- Non-Elitist Pareto-based Methods
  - Pure Pareto ranking
  - MOGA
  - NSGA
  - NPGA and NPGA 2

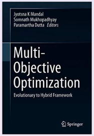
A Taxonomy of MOEAs



Contemporary Approaches

- Elitist Pareto-based Methods
  - SPEA and SPEA2
  - NSGA-II
  - PAES, PESA and PESA II
  - Micro-genetic Algorithm for Multi-Objective Optimization and μGA^2
  - Many others (most of them already forgotten...)

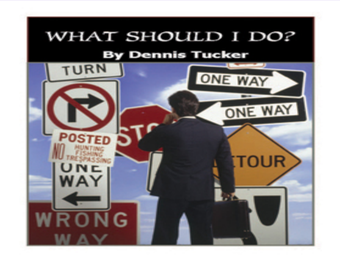
A Taxonomy of MOEAs



Recent Approaches

- MOEA/D (and its many variants)
- Indicator-Based Approaches
  - IBEA
  - SMS-EMOA
  - HyPE
  - Other Approaches
- NSGA-III (and its many variants)

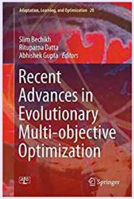
Where are we heading?



Introduction

After 38 years of existence, and with so much work done, EMO may seem intimidating to some people. If so many people have worked in this area for the last 15 years, what remains to be done?

Recent Results in Algorithms



Algorithms

As indicated before, there are three main types of MOEAs in current use:

- Pareto-based MOEAs
- Decomposition-based MOEAs
- Indicator-based MOEAs

Recent Results in Algorithms

Pareto-based MOEAs

These are the traditional MOEAs in which the selection mechanism is based on Pareto optimality. Most of them adopt some form of nondominated sorting and a density estimator (e.g., crowding, fitness sharing, entropy, adaptive grids, parallel coordinates, etc.).

Main limitations

Scalability in objective function space is clearly a limitation of Pareto-based MOEAs unless a significantly larger population size is adopted. Another alternative is to change the density estimator, but most people don't seem to be interested in moving in that direction.

Recent Results in Algorithms

Decomposition-based MOEAs

The core idea of these approaches is to transform a multi-objective problem into several single-objective optimization problems which are simultaneously solved using information from its neighboring subproblems.

Main limitations

The performance of decomposition-based MOEAs relies on the scalarizing function that they adopt. They are also sensitive to the method used to generate weights. However, they are scalable in objective function space (although an increase in the number of objectives will increase the population size).

Recent Results in Algorithms

Indicator-based MOEAs

The original idea was to adopt a performance indicator for the selection mechanism of a MOEA. However, some researchers discovered that the mere use of a performance indicator in the density estimator was enough to have a good performance (e.g., SMS-EMOA).

Main limitations

The only performance indicator which is known to be Pareto compliant is the hypervolume, which is computationally expensive in high dimensionality (in objective space). Other performance indicators are available, some of which are weakly Pareto compliant (e.g., $R2$ and IGD+). However, researchers don't seem to like them much.

Recent Results in Algorithms

A common practice of today's research

Based on the contents of most of the papers that I normally read in top journals (e.g., IEEE TEC) regarding the design of new MOEAs, today most researchers propose "new" algorithmic variants based on the existing benchmarks (ZDT, DTLZ, WFG, UF, etc.).

Where are the new ideas?

We can of course keep the current trend of producing many variants of the most popular MOEAs in current use (i.e., MOEA/D and NSGA-III), but are we really heading somewhere with this? Can we design MOEAs based on a different idea or at least be more creative regarding the enhancements that we propose to the existing MOEAs?

Recent Results in Algorithms

Here is an idea

Some years ago (at EMO'2015), we proposed an approach that transforms a multi-objective optimization problem into a linear assignment problem using a set of weight vectors uniformly scattered. Uniform design is adopted to obtain the set of weights, and the Kuhn-Munkres (Hungarian) algorithm is used to solve the resulting assignment problem. This approach was found to perform quite well (and at a low computational cost) in many-objective optimization problems. This approach does not belong to any of the three types of MOEAs that I previously indicated. An improved version of this algorithm was recently published.

Luis Miguel Antonio, José A. Molinet Berenguer and Carlos A. Coello Coello, "**Evolutionary Many-objective Optimization based on Linear Assignment Problem Transformations**", *Soft Computing*, Vol. 22, No. 6, pp. 5491–5512, August 2018.

Recent Results in Algorithms

What else can we do?

I believe that it's very important to understand the limitations of current MOEAs. For example, knowing that some scalarizing functions offer advantages over others is very useful to design good decomposition-based and even indicator-based MOEAs (MOEAs based on $R2$ normally rely on decomposition).

Miriam Pescador-Rojas, Raquel Hernández Gómez, Elizabeth Montero, Nicolás Rojas-Morales, María-Cristina Riff and Carlos A. Coello Coello, "**An Overview of Weighted and Unconstrained Scalarizing Functions**", in Heike Trautmann, Günter Rudolph, Kathrin Klamroth, Oliver Schütze, Margaret Wiecek, Yaochu Jin and Christian Grimme (Editors), *Evolutionary Multi-Criterion Optimization, 9th International Conference, EMO 2017*, pp. 499–513, Springer, Lecture Notes in Computer Science Vol. 10173, Münster, Germany, March 19-22, 2017, ISBN 978-3-319-54156-3.

Recent Results in Algorithms

What else can we do?

Another intriguing idea is the combination of different MOEAs under a single control mechanism (e.g., AMALGAM [Vrugt and Robinson, 2007]).

Jasper A. Vrugt and Bruce A. Robinson, "**Improved evolutionary optimization from genetically adaptive multimethod search**", *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 104, No. 3, pp. 708–711, January 16, 2007.

Recent Results in Algorithms

What else can we do?

Another interesting idea is to combine components of MOEAs under a single framework that allows to exploit their advantages. This is the basic idea of **Borg** [Hadka & Reed, 2013], which adopts ϵ-dominance, a measure of convergence speed called ϵ progress, an adaptive population size, multiple recombination operators and a steady-state selection mechanism.

Could this lead to the automated design of MOEAs as has been suggested by researchers from automated parameter tuning for single-objective EAs (e.g., Thomas Stützle)?

David Hadka and Patrick Reed, “**Borg: An Auto-Adaptive Many-Objective Evolutionary Computing Framework**, *Evolutionary Computation*, Vol. 21, No. 2, pp. 231–259, Summer 2013.

Scalability

Many-Objectivity

MOEAs that adopt a selection mechanism based on Pareto optimality do not scale properly: the number of nondominated solutions grows exponentially with the number of objectives [Farina, 2004]. This makes the selection mechanism completely useless, since all the nondominated solutions are considered equally good!

M. Farina and P. Amato, "**A fuzzy definition of "optimality" for many-criteria optimization problems**", *IEEE Transactions on Systems, Man, and Cybernetics Part A–Systems and Humans*, Vol. 34, No. 3, pp. 315–326, May 2004.

Scalability

Many-Objectivity

In the early days of this area, two types of approaches were commonly adopted to cope with many-objective problems:

- Adopt or propose a preference relation that induces a finer grain order on the solutions than that induced by the Pareto dominance relation [Di Pierro, 2007; Farina, 2002; Sülflow, 2007; Sato, 2007].
- To reduce the number of objectives of the problem during the search process [Brockhoff, 2006], or *a posteriori*, during the decision making process [Deb, 2006; Brockhoff, 2007; Lopez Jaimes, 2008].

Many other approaches are possible, including, for example, the use of machine learning techniques (as in MONEDA [Martí et al., 2008]), performance indicators (as in SMS-EMOA [Beume, 2007]), ϵ-dominance or the two-archive MOEA, which uses one archive for convergence and another for diversity [Praditwong & Yao, 2006].

Scalability

Many-Objectivity

It has been empirically shown that a random search is more effective than the NSGA-II when dealing with more than 10 objectives [Mostaghim & Schmeck, 2008].

There is also another interesting problem related to scalability: as we increase the number of objectives, the number of solutions required to sample the Pareto front, also grows exponentially.

Sanaz Mostaghim and Hartmut Schmeck, "**Distance Based Ranking in Many-Objective Particle Swarm Optimization**", in Günter Rudolph, Thomas Jansen, Simon Lucas, Carlo Poloni and Nicola Beume (Editors), *Parallel Problem Solving from Nature–PPSN X*, pp. 753–762, Springer. Lecture Notes in Computer Science Vol. 5199, Dortmund, Germany, September 2008.

Scalability

Many-Objectivity

But, what is the source of difficulty in many-objective problems? Ishibuchi et al. [2015] considered 5 types of difficulties that arise in many-objective problems, including the typical ones (e.g., difficulties to generate a good approximation of the entire Pareto front) and others that are not so obvious (e.g., difficulties to assess performance).

Hisao Ishibuchi, Naoya Akedo and Yusuke Nojima, "**Behavior of Multiobjective Evolutionary Algorithms on Many-Objective Knapsack Problems**", *IEEE Transactions on Evolutionary Computation*, Vol. 19, No. 2, pp. 264–283, April 2015.

Scalability

Many-Objectivity

Schütze et al. [2011] had already studied the actual source of difficulty in many-objective problems. They concluded that adding more objectives for a problem doesn't necessarily makes it harder. Also, Ishibuchi et al. [2015] showed that NSGA-II could properly solve many-objective knapsack problems in which the objectives were highly correlated.

Oliver Schütze, Adriana Lara and Carlos A. Coello Coello, "**On the Influence of the Number of Objectives on the Hardness of a Multi-Objective Optimization Problem**", *IEEE Transactions on Evolutionary Computation*, Vol. 15, No. 4, pp. 444–455, August 2011.

Scalability

Many-Objectivity

There are, however, many other issues related to many-objective optimization that deserve to be studied. For example:

- Density estimators (what to use and what sort of distributions do we aim to find?)
- Visualization of high-dimensional Pareto fronts (see for example Tušar and Filipič [2015]).
- Performance indicators suitable for assessing performance (not as expensive as the hypervolume).

Tea Tušar and Bogdan Filipič, "**Visualization of Pareto Front Approximations in Evolutionary Multiobjective Optimization: A Critical Review and the Prosection Method**", *IEEE Transactions on Evolutionary Computation*, Vol. 19, No. 2, pp. 225–245, April 2015.

Scalability

What about scalability in decision variable space?

Until a few years ago, there is almost no work on this topic, which also deserves attention. Zhang & Lim [2007] performed a small study with a single problem that was scaled up to 100 decision variables. Durillo et al. [2008] did a more thorough study, adopting from 8 to 2048 decision variables. Perhaps the most remarkable finding was that PAES was the most salient technique from the several compared (NSGA-II, SPEA2, MOCell, OMOPSO and PESA-II). OMOPSO did very well up to 256 decision variables and ranked second between 512 and 1024 decision variables.

Juan J. Durillo, Antonio J. Nebro, Carlos A. Coello Coello, Francisco Luna and Enrique Alba, "**A Comparative Study of the Effect of Parameter Scalability in Multi-Objective Metaheuristics**", in *2008 IEEE Congress on Evolutionary Computation (CEC'2008)*, pp. 1893–1900, IEEE Service Center, Hong Kong, June 2008.

Scalability

What about scalability in decision variable space?

Antonio and Coello Coello [2013] proposed the first MOEA explicitly designed for large-scale multi-objective optimization. This MOEA is based on cooperative coevolution and deal solve problems having up to 5,000 decision variables.

Luis Miguel Antonio and Carlos A. Coello Coello, "**Use of Cooperative Coevolution for Solving Large Scale Multiobjective Optimization Problems**", in *2013 IEEE Congress on Evolutionary Computation (CEC'2013)*, pp. 2758–2765, IEEE Press, Cancún, México, 20-23 June, 2013.

Scalability

What about scalability in decision variable space?

Although other MOEAs have been proposed since 2013 for large scale multi-objective optimization, several topics remain to be explored in this area. For example, we don't have a set of test problems that had been explicitly designed for testing large scale MOEAs.

What about large scale many-objective optimization?

Can we deal with large scale many-objective optimization problems? What sort of MOEA would be required in this case? See for example Cao et al. [2017].

Bin Cao, Jianwei Zhao, Zhihan Lv, Xin Liu, Shan Yang, Xinyuan Kang and Kai Kang, "**Distributed Parallel Particle Swarm Optimization for Multi-Objective and Many-Objective Large-Scale Optimization**", *IEEE Access*, Vol. 5, pp. 8214–8221, 2017.

Parallelism

Why do we need Parallel MOEAs?

Evidently, the main motivation to develop parallel MOEAs is to deal with expensive objective functions, which are common in real-world applications. However, something surprising is that the design of new parallel MOEAs is very rare in the specialized literature.

El-Ghazali Talbi, Sanaz Mostaghim, Tatsuya Okabe, Hisao Ishibuchi, Günter Rudolph and Carlos A. Coello Coello, "**Parallel Approaches for Multi-objective Optimization**", in Jürgen Branke, Kalyanmoy Deb, Kaisa Miettinen and Roman Slowinski (Editors), *Multiobjective Optimization. Interactive and Evolutionary Approaches*, pp. 349–372, Springer. Lecture Notes in Computer Science Vol. 5252, Berlin, Germany, 2008.

Parallelism

What else can we do regarding Parallel MOEAs?

We are lacking work on the development of asynchronous parallel MOEAs and their comparison with respect to their synchronous counterparts (see for example Wessing et al. [2016]).

Simon Wessing, Günter Rudolph and Dino A. Menges, "**Comparing Asynchronous and Synchronous Parallelization of the SMS-EMOA**", in Julia Handl, Emma Hart, Peter R. Lewis, Manuel López-Ibáñez, Gabriela Ochoa and Ben Paechter, *Parallel Problem Solving from Nature – PPSN XIV, 14th International Conference*, pp. 558–567, Springer. Lecture Notes in Computer Science Vol. 9921, Edinburgh, UK, September 17-21, 2016.

Parallelism

What else can we do regarding Parallel MOEAs?

We also need parallel MOEAs designed to run on GPUs (see for example Zhu et al. [2011]).

Weihang Zhu, Ashraf Yaseen and Yaohang Li, "**DEMCMC-GPU: An Efficient Multi-Objective Optimization Method with GPU Acceleration on the Fermi Architecture**", *New Generation Computing*, Vol. 29, No. 2, pp. 163–184, 2011.

Parallelism

What else can we do regarding Parallel MOEAs?

Another interesting topic is the change of granularity in a parallel MOEA (with a unidirectional topology) with the aim of performing a more efficient search (see for example López Jaimes and Coello [2007]).

Antonio López Jaimes and Carlos A. Coello Coello, "**MRMOGA: A New Parallel Multi-Objective Evolutionary Algorithm Based on the Use of Multiple Resolutions**", *Concurrency and Computation: Practice and Experience*, Vol. 19, No. 4, pp. 397–441, March 25, 2007.

Parallelism

What else can we do regarding Parallel MOEAs?

There are many other possibilities. For example, the use of asynchronous parallelism combined with the use of micro-populations, as done in S-PAMICRO (PArallel MICRo Optimizer based on the S metric). This is a computationally efficient (parallel) version of SMS-EMOA that uses exact hypervolume contributions (see [Hernández Gómez and Coello [2016]).

Raquel Hernández-Gómez, Carlos A. Coello Coello and Enrique Alba, "**A Parallel Version of SMS-EMOA for Many-Objective Optimization Problems**", in Julia Handl, Emma Hart, Peter R. Lewis, Manuel López-Ibáñez, Gabriela Ochoa and Ben Paechter (Editors), *Parallel Problem Solving from Nature – PPSN XIV, 14th International Conference*, pp. 568–577, Springer. Lecture Notes in Computer Science Vol. 9921, Edinburgh, UK, September 17-21, 2016.

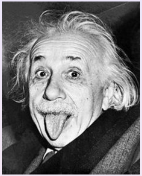
Parallelism

What else can we do regarding Parallel MOEAs?

I believe that a crucial task is to try to exploit parallel architectures by designing MOEAs that explicitly take advantage of a particular architecture (e.g., grid computing or GPUs) rather than simply producing ad-hoc parallel versions of existing MOEAs such as MOEA/D and NSGA-II.

There are, however, many other research topics worth exploring in this area. For example, theoretical analysis of MOEAs, fitness landscape analysis of parallel MOEAs, the impact of the topology on the performance of a parallel MOEA, automated parameter tuning of MOEAs, decomposition-based parallel MOEAs, indicator-based parallel MOEAs, and diversity management in parallel MOEAs, among many others.

Some Challenges



Clearly, the main challenge for the coming years is to continue to open new venues of research in EMOO. This is becoming increasingly difficult, given the huge volume of research that has been conducted and is currently ongoing around the world. So, we need to be more creative!

Imagination is more important than knowledge.

Albert Einstein

Some Challenges

Personally, I believe that there are still plenty of topics to study within EMOO, but some of them require us to move outside the main stream of the research that is currently being conducted. For example, we need new performance indicators, particularly for many-objective optimization. We don't have appropriate performance indicators for assessing diversity in many-objective optimization, although there are some interesting choices (e.g., *s*-energy).

D.P. Hardin and E.B. Saff, "**Discretizing Manifolds via Minimum Energy Points**", *Notices of the American Mathematical Society*, pp. 1186–1194, Vol. 51, No. 10, November 2004.

Some Challenges

It is also important to design new mechanisms (e.g., operators, encodings, etc.) for MOEAs based on specific features of real-world problems (e.g., variable length encodings, expensive objective functions, uncertainty, etc.). See for example Li & Deb [2017].

Hui Li and Kalyanmoy Deb, "**Challenges for Evolutionary Multiobjective Optimization Algorithms in Solving Variable-Length Problems**", in *2017 IEEE Congress on Evolutionary Computation (CEC'2017)*, pp. 2217–2224, IEEE Press, San Sebastián, Spain, June 5-8, 2017.

Some Challenges

The way in which coevolutionary approaches can help us to solve complex multi-objective optimization problems is another interesting venue for future research. Besides large scale problems, coevolution can help us in other domains (e.g., dynamic multi-objective optimization problems [Goh and Tan, 2009]), but its potential has been scarcely studied in this area (see [Miguel Antonio & Coello Coello, 2018]).

Chi-Keong Goh and Kay Chen Tan, "**A Competitive-Cooperative Coevolutionary Paradigm for Dynamic Multiobjective Optimization**", *IEEE Transactions on Evolutionary Computation*, Vol. 13, No. 1, pp. 103–127, February 2009.

Luis Miguel Antonio and Carlos A. Coello Coello, "**Coevolutionary Multi-objective Evolutionary Algorithms: A Survey of the State-of-the-Art**", *IEEE Transactions on Evolutionary Computation*, in press.

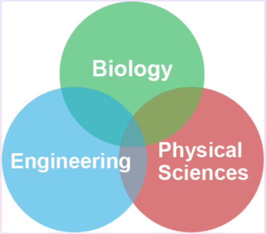
Some Challenges

We also need to explore more ways of bridging the gap between Operations Research and EMOO. An example is the development of hybrid approaches that combine a MOEA with a mathematical programming technique (see for example Lara et al. [2010]). Another one is the use of the Karush-Kuhn-Tucker optimality conditions to estimate proximity of a solution to the Pareto optimal set (see Abouhawwash et al. [2017]).

Adriana Lara, Gustavo Sanchez, Carlos A. Coello Coello and Oliver Schütze, "**HCS: A New Local Search Strategy for Memetic Multi-Objective Evolutionary Algorithms**", *IEEE Transactions on Evolutionary Computation*, Vol. 14, No. 1, pp. 112–132, February 2010.

Mohamed Abouhawwash, Haitham Seada and Kalyanmoy Deb, "**Towards Faster Convergence of Evolutionary Multi-Criterion Optimization Algorithms Using Karush Kuhn Tucker Optimality Based Local Search**", *Computers & Operations Research*, Vol. 79, pp. 331–346, March 2017.

Some Challenges



However, it's important to keep in mind that a great source of diversity regarding research ideas is the knowledge coming from other disciplines. For example, our field has adopted advanced data structures (e.g., red-black trees), concepts from computational geometry (e.g., convex hulls, quadtrees and Voronoi maps), and from economics (e.g., game theory) to design novel MOEAs and operators.

Some Challenges

Summarizing, I am convinced that EMOO is still a very promising research area which should remain active for many more years (or so I hope). However, we need to increase diversity in our research topics and to be more disruptive. If we only do work by analogy, we will suffer stagnation!

To know more about evolutionary multi-objective optimization

Please visit the new webpage of the EMOO repository:

https://emoo.cs.cinvestav.mx/

To know more about evolutionary multi-objective optimization

The EMOO repository currently contains:

- Over 13,430 bibliographic references including 307 PhD theses, 53 Masters theses, over 6878 journal papers and over 4560 conference papers.
- Contact information of 79 EMOO researchers.
- Public domain implementations of SPEA, NSGA, NSGA-II, the microGA, MOGA, ϵ-MOEA, MOPSO and PAES, among others.