Regularities and dynamics in bisimulation reductions of big graphs

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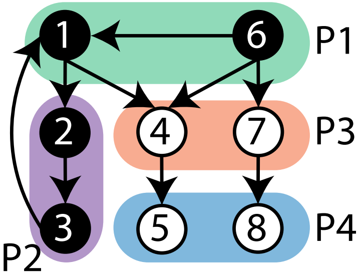
Outline

Motivation

Experimental setup

Results

Insights



An example of bisimulation reduction

Bisimulation reduction

- Bisimulation partitioning is an important concept in many fields (computer science, modal logic, etc.), in DB research as well (structural index, graph reduction)
- It can be seen as a way of clustering nodes

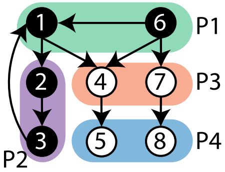


Figure: Bisimulation partition example, partition block graph (reduction graph) $\{P2 \leftrightarrow P1 \rightarrow P3 \rightarrow P4\}$

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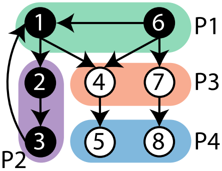

Figure: Bisimulation partition example, partition block graph (reduction graph) $\{P2 \leftrightarrow P1 \rightarrow P3 \rightarrow P4\}$

- Reduce graph size while preserving structural properties (e.g., reachability)
- Result can be seen as a graph
- Many algorithms, no work on analyzing the results

Questions

Regularities, such as power-law distribution exists in real graphs.

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- Do graphs under bisimulation reduction also have such properties?
- How would that knowledge help us?

Experimental setup for investigation

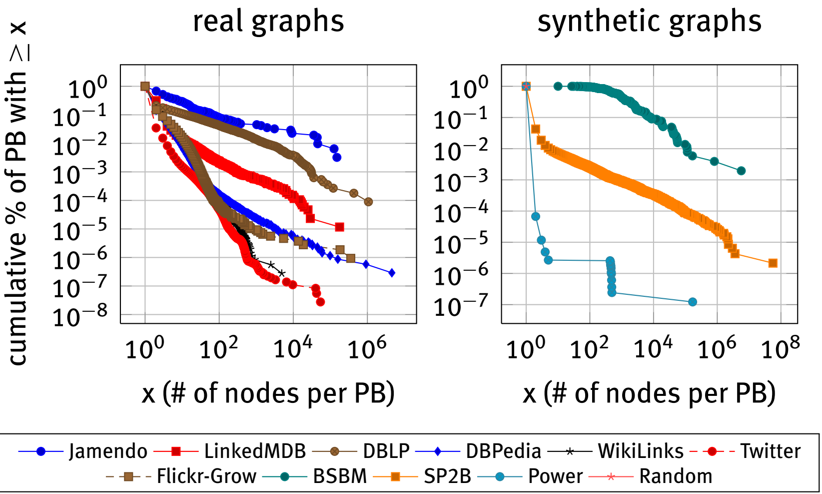
- Big graphs, from 1 Million to 1.4 Billion edges (Twitter, DBPedia, etc.)
- One dynamic social graph, from 17 Million to 33 Million edges (Flickr-grow)
- State-of-the-art I/O efficient algorithm for computing bisimulation reductions (k-bisim, $k = 10$)
- We use cumulative distribution function (CDF) to present distributions

Regularities - bisimulation result

Power-law also exists in many attributes for bisimulation partition results for *real graphs*. But this is not the case for *synthetic graphs*.

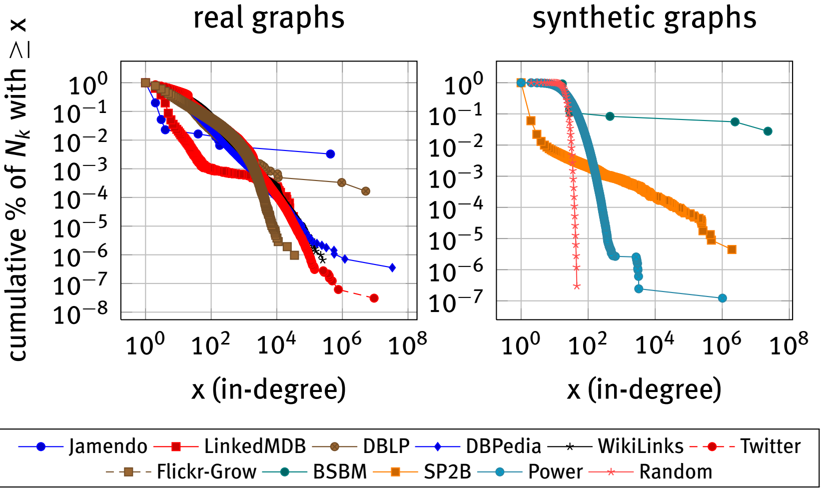
Regularities - bisimulation result

Partition block size distribution



Regularities - bisimulation result

Bisimulation graph in/out-degree distribution



Dynamics - a real growing social graph

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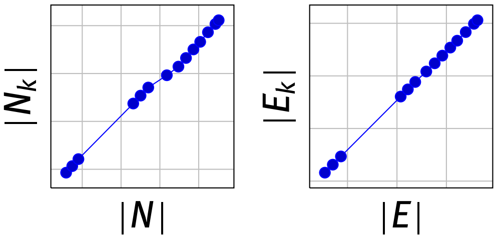
Dynamics - a real growing social graph

- Does the bisimulation result grow when the original graph grows?
  - Yes.

- How fast does it grow?
  - Linearly with respect to the original graph.



Insights

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- Power-law distributions in bisimulation results $\Rightarrow$ skew expected in applications (indexes, data partitioned among machines, ...)
- Behaviors of graph generators $\Rightarrow$ some more work needs to be done for graph generators
- Bisimulation result/graph grows $\Rightarrow$ lower k or other adaptations (e.g., choose different k for different parts of the graph, different node/edge labeling)

Thank you! Q&A

For more information, just google *seeqr project*
or visit: `bit.ly/seeqr`

Definition of k-*bisimilar*

Definition

Let k be a non-negative integer and $G = \langle N, E, \lambda_N, \lambda_E \rangle$ be a graph. Nodes $u, v \in N$ are called k-*bisimilar* (denoted as $u \approx^k v$), iff the following holds:

1. $\lambda_N(u) = \lambda_N(v)$,
2. if $k > 0$, then for any edge $(u, u') \in E$, there exists an edge $(v, v') \in E$, such that $u' \approx^{k-1} v'$ and $\lambda_E(u, u') = \lambda_E(v, v')$, and
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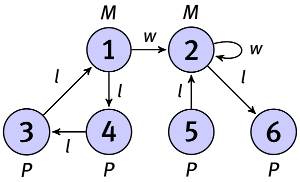
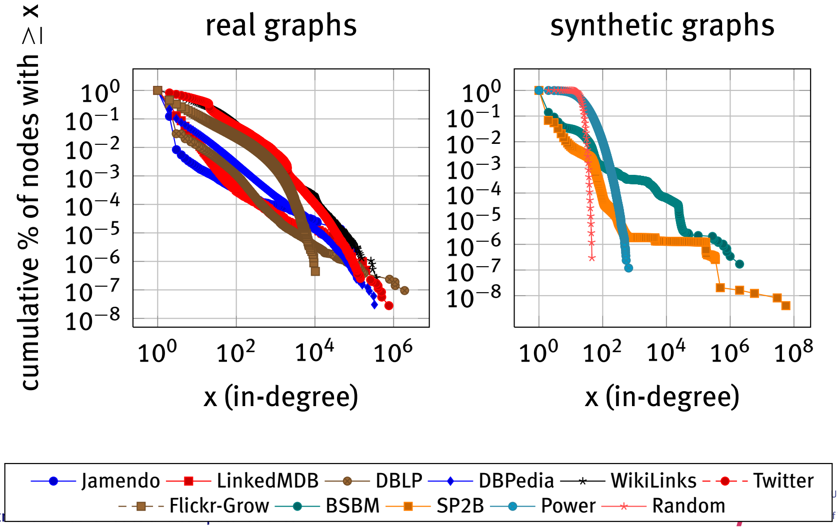


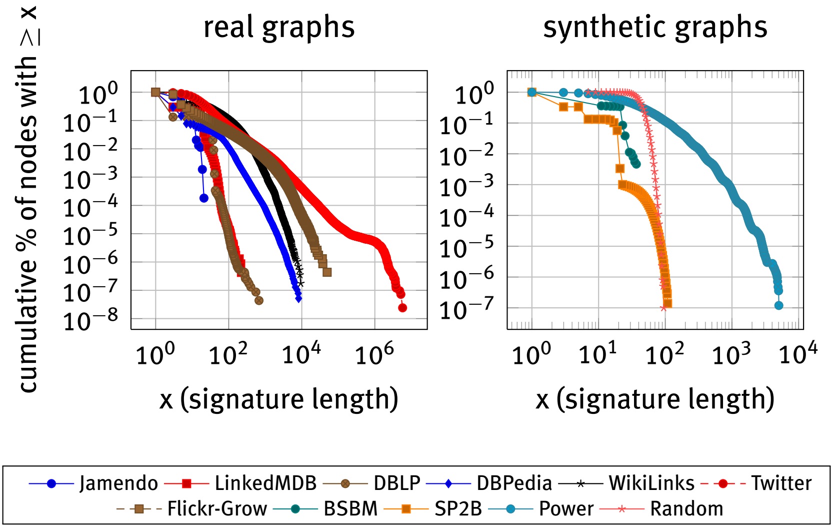
In this example graph, nodes 1 and 2 are 0- and 1- bisimilar but not 2-bisimilar.

Regularities - original graphs

Power-law exists in in/out-degree distribution for most of the examined graphs.



Signature length



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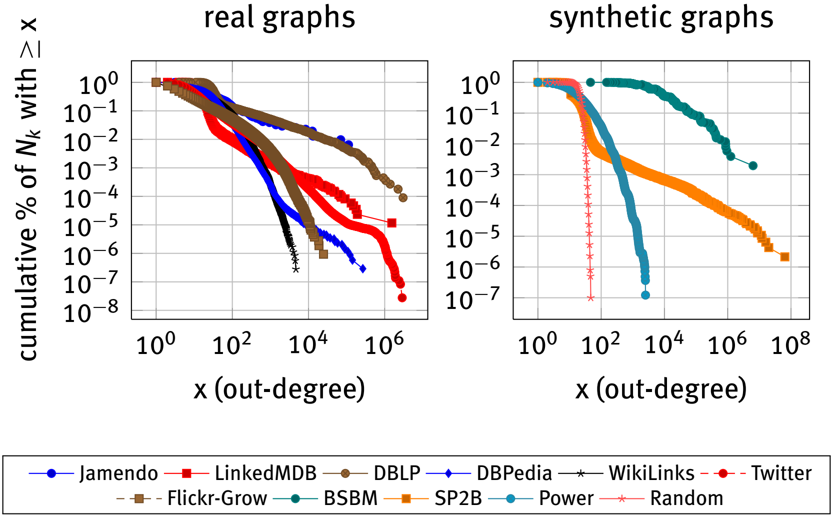
Out-degree


real graphs
synthetic graphs
cumulative % of N_k with $\geq$ x
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^0
10^2
10^4
10^6
10^8
x (out-degree)
x (out-degree)
Jamendo
LinkedMDB
DBLP
DBPedia
WikiLinks
Twitter
Flickr-Grow
BSBM
SP2B
Power
Random