Optimal and Adaptive Algorithms for Online Boosting

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- At the end, predict by taking a (weighted) majority vote.

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An natural question: how to extend boosting to the online setting?

Related Work

Several algorithms exist (Oza and Russell, 2001; Grabner and Bischof, 2006; Liu and Yu, 2007; Grabner et al., 2008).

- mimic offline counterparts.
- achieve great success in many real-world applications.
- no theoretical guarantees.

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Chen et al. (2012): first online boosting algorithms with theoretical guarantees.

- online analogue of weak learning assumption.
- connecting online boosting and smooth batch boosting.

Batch Boosting

Given a batch of T examples, $(\mathbf{x}_t, y_t) \in \mathcal{X} \times \{-1, 1\}$ for $t = 1, \ldots, T$.
Learner $\mathcal{A}$ predicts $\mathcal{A}(\mathbf{x}_t) \in \{-1, 1\}$ for example $\mathbf{x}_t$.

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$\Downarrow$ Boosting (Schapire, 1990; Freund, 1995)

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Weak Online learner $\mathcal{A}$ (with edge γ and excess loss S):

$$\sum_{t=1}^{T} \mathbf{1}\{\mathcal{A}(\mathbf{x}_t) \neq y_t\} \leq (\tfrac{1}{2} - \gamma)T + S$$

Strong Online learner $\mathcal{A}'$ (with any target error rate ϵ and excess loss S')

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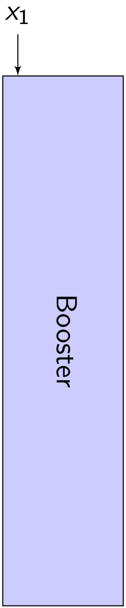
this talk: $S = \frac{1}{\gamma}$ (corresponds to $\sqrt{T}$ regret)

Main Results

Parameters of interest:
N = number of weak learners (of edge γ) needed to achieve error rate ϵ.
T_ϵ = minimal number of examples s.t. error rate is ϵ.

Algorithm	N	T_ϵ	Optimal?	Adaptive?
Online BBM	$O(\frac{1}{\gamma^2} \ln \frac{1}{\epsilon})$	$\tilde{O}(\frac{1}{\epsilon\gamma^2})$	$\surd$	$\times$
AdaBoost.OL	$O(\frac{1}{\epsilon\gamma^2})$	$\tilde{O}(\frac{1}{\epsilon^2\gamma^4})$	$\times$	$\surd$
Chen et al. (2012)	$O(\frac{1}{\epsilon\gamma^2})$	$\tilde{O}(\frac{1}{\epsilon\gamma^2})$	$\times$	$\times$

Structure of Online Boosting



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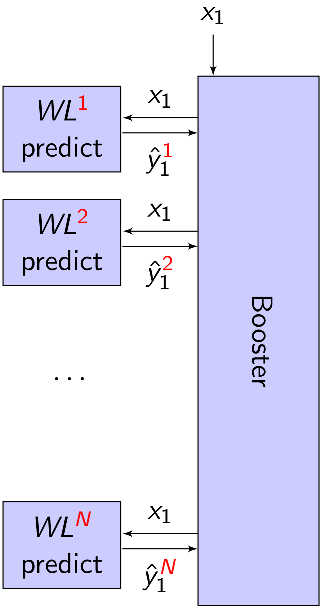

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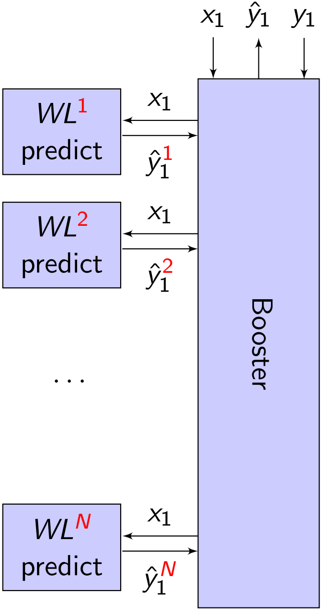

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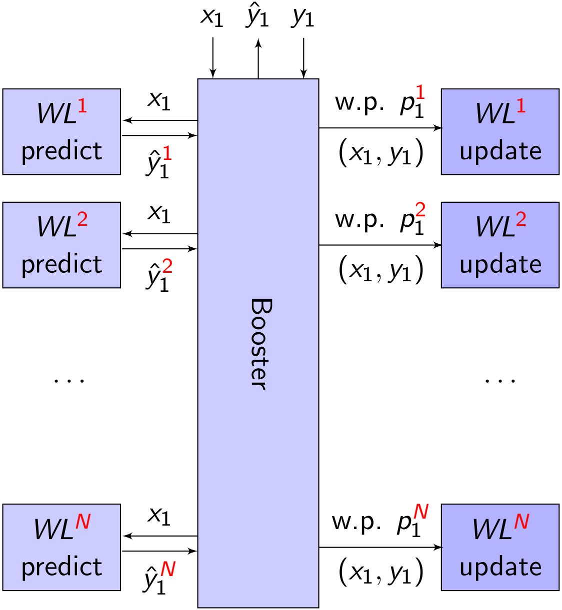

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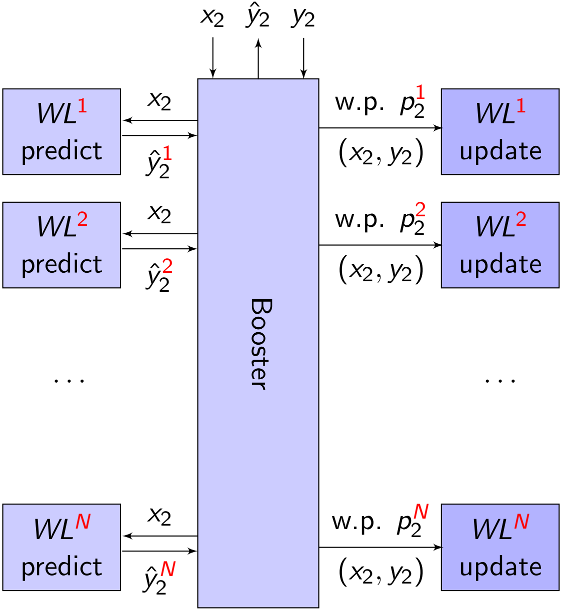

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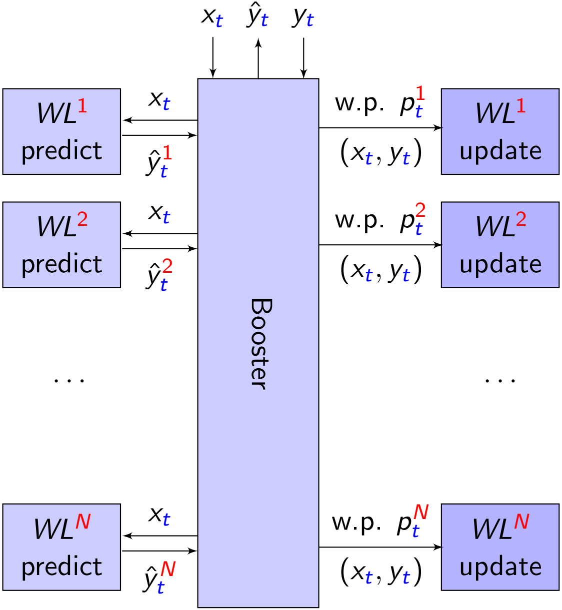

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Online version: sequence of potentials $\Phi_i(s)$ s.t.

$$\Phi_N(s) \geq \mathbf{1}\{s \leq 0\},$$
$$\Phi_{i-1}(s) \geq (\tfrac{1}{2} - \tfrac{\gamma}{2})\Phi_i(s-1) + (\tfrac{1}{2} + \tfrac{\gamma}{2})\Phi_i(s+1).$$

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Online boosting algorithm using Φ_i:

- prediction: majority vote.
- update: $p_t^i = \Pr[(\mathbf{x}_t, y_t) \text{ sent to } i\text{th weak learner}] \propto w_t^i$ where $w_t^i =$ difference in potentials if example is misclassified or not.

Mistake Bound

Generalized drifting games analysis implies

$$\sum_{t=1}^{T} \mathbf{1}\{\mathcal{A}'(\mathbf{x}_t) \neq y_t\} \leq \underbrace{\Phi_0(0)}_{\leq \epsilon} T \; + \; \underbrace{(S + \tfrac{1}{\gamma}) \sum_i \|\mathbf{w}^i\|_\infty}_{=S'} .$$

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Online BBM: to get ϵ error rate, needs
$N = O(\frac{1}{\gamma^2} \ln(\frac{1}{\epsilon}))$ weak learners and $T_\epsilon = O(\frac{1}{\epsilon\gamma^2})$ examples. (Optimal)

Drawback of Online BBM

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Adaptivity is the key advantage of AdaBoost!

- different weak learners weighted differently based on their performance.
- Adapts to *easy data*

Adaptivity via Online Loss Minimization

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We generalize this to the online setting:

- replace line search with online gradient descent.
- exponential loss does not work again, use logistic loss to get adaptive online boosting algorithm AdaBoost.OL.

Intuition and main ideas

- Classifier f with real-valued output $f(x)$: predict $\operatorname{sign}(f(x))$
- Logistic loss $\ln(1 + \exp(-f(x)y))$: surrogate for $1\{\operatorname{sign}(f(x)) \neq y\}$

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- for each i, search for step-size α using *online gradient descent* over sequence of T data points
- for each data point, final prediction is weighted majority with weights given by current α's

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If WL^i has edge γ_i, then

$$\sum_{t=1}^{T} \mathbf{1}\{\mathcal{A}'(\mathbf{x}_t) \neq y_t\} \leq \frac{2T}{\sum_i \gamma_i^2} + \tilde{O}\left(\frac{N^2}{\sum_i \gamma_i^2}\right)$$

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Not optimal but adaptive.

Results

Available in Vowpal Wabbit 8.0.

- command line option: `--boosting`.
- VW as the default "weak" learner (a rather strong one!)

Dataset	VW baseline	Online BBM	AdaBoost.OL	Chen et al. 12
20news	0.0812	0.0775	0.0777	0.0791
a9a	0.1509	0.1495	0.1497	0.1509
activity	0.0133	0.0114	0.0128	0.0130
adult	0.1543	0.1526	0.1536	0.1539
bio	0.0035	0.0031	0.0032	0.0033
census	0.0471	0.0469	0.0469	0.0469
covtype	0.2563	0.2347	0.2495	0.2470
letter	0.2295	0.1923	0.2078	0.2148
maptaskcoref	0.1091	0.1077	0.1083	0.1093
nomao	0.0641	0.0627	0.0635	0.0627
poker	0.4555	0.4312	0.4555	0.4555
rcv1	0.0487	0.0485	0.0484	0.0488
vehv2binary	0.0292	0.0286	0.0291	0.0284

Conclusions

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Open problem: optimal and adaptive algorithm?